University of Alberta

Silence and Voice
in the
Secondary Mathematics Classroom

by

David Wagner ©

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fulfillment of the requirements for the degree of Doctor of Philosophy

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Chapter 1 – Prelude

In a mathematics classroom, language counts. Every classroom gives rise to a complex web of relationships among students, their teacher and powerful traditions within the school, community and culture, yet the discourse that forms the connections in this web is typically left unquestioned within the classroom itself. In the pages that follow, I will describe my conversations with a group of 15- and 16-year olds who came together to learn the mathematics prescribed for them in a grade 11 pure mathematics class.[1] In this conversation, I tried to raise their awareness of their language practice in their classroom by looking together with them at transcripts of their own discourse.

I wondered what critical language awareness (CLA) would do for these students' experience of mathematics, or perhaps what it would do *to* their experience of mathematics. There is a growing interest in including CLA in high school curricula, mostly in English language arts and in second language acquisition courses. I find CLA absent from published accounts of mathematics curricula. My interest, focused as a single question, could be written as follows.

> Considering the depth and breadth of students' mathematical experience, what is the effect of their mediated use of discourse analytic tools to explore the mathematics discourse that surrounds them?

Before describing the particular outcomes of my motivated conversations with the classroom participants, I will give an account of factors that I find significant in my desire to engage with this particular project. This background is an important part of the account, because it is the story I brought into the conversation.

I present this account of the research in a narrative form in recognition of the complexity and richness of the researched classroom and the multiple forms an account of our conversation could have taken. However, the *research* itself was not done in the tradition of narrative inquiry, which is described well by Clandinin and Connelly (2000). Rather, the *form* of this report is structured by a narrative frame.

Orienting Background

After teaching mathematics for five years in Alberta, I lived for two and a half years in community with rural teachers in Swaziland along with my wife Carolyn and our two daughters. As I taught locally and collaborated with the region's teachers, I became more aware of the dangers inherent in teaching and learning that do not include critical reflection. I also realized that I had not reflected critically on my pedagogy while I taught in Alberta, "at home." In Swaziland, I found it easier to see a lack of congruence between practice and professed values when looking at a culture foreign to me. Furthermore, as I became more critically aware in the Swazi culture, I was able to think more critically about my home culture even though I was not directly experiencing it at the time. When we returned to Alberta, our home culture seemed foreign to us, probably because our worldviews had changed considerably through our Swazi experience. Here at home, informed by a new interest in reflective practice, I could not avoid thinking critically about many cultural practices, including my professional practice as a teacher. I found that in order to reflect critically, it helped to look at my practice as though it were strange. My time away from home made it easier for me to see home practices as strange.

In my first year back in Alberta, I led the senior high mathematics teachers in my school district through their struggles with the province's significantly new program of studies. A new set of courses was entering the mainstream – Applied Mathematics. Because of the re-visioning and realignment of mathematics teaching practice in this province, my critical reflection focused primarily on the mathematics classroom as a cultural site, though I also reflected critically on other educational practices. After one year teaching "back home," I decided to pursue full-time graduate studies to focus my energies on this reflection.

In both my master's research and my research assistantship, I explored mathematics classroom tasks that would prompt students to communicate with each other and turn their attention to the mathematical thinking of their peers (Wagner, 2002a and 2003c; Jackson and Wagner, in press). In my master's research, I observed two Pure Mathematics 10 classes[2] engaging in some open-ended investigative projects I had developed. Although most participating students revelled in the chance to think and communicate about real problems within mathematics itself, they and their teachers were also bewildered and unsure of how to interact in a mathematical context that was somewhat foreign to them. I watched them wonder what was expected of them and struggle to find words for their mathematical ideas and questions. In this instance, mathematics itself was made strange for them.

Building on this revelation from my master's research, I have become interested in exploring ways in which students and teachers can become more aware of the relationships in their mathematics classroom and discover connections between their use of language and their mathematical thinking. I am thinking both about pedagogical relationships and about relationships within the evolving structures of mathematics. My principal concern, at least for this study, is for the relationships that pertain to mathematics.

At first, I had no explanation for my fascination with mathematics tasks such as the ones I was exploring. Eventually I came to recognize that I wanted to investigate classroom possibilities that mathematics teachers could incorporate every day and that could, at the same time, prompt students to direct their attention to each other in addition to the mathematics curriculum content. Assuming that the way we enculture students in mathematics classes "formats" the way they address problems outside the classroom, an assumption supported by Skovsmose (2000) who coined this term as a way of describing the impact of mathematics education on society, this effort to promote tasks that direct students' attention to each other's thinking could make a positive impact on the world by formatting young people to be more mindful of others.

For my activity to seem to me a worthwhile direction of energy, attention and creativity, I need to see that it contributes to a healthy world – a world in which people live peaceably with each other, treating each other as equals. I wrote of this aspiration for an audience of teachers:

> Although I value mathematics education, I consider the quest for peace my vocation. In the mathematics education context, I can work at bringing peace to my world. I recognize that realizing my goal of a peaceful world in my lifetime is unlikely. Perhaps it is the mathematician in me who is content to approach the inaccessible, the infinite. (Wagner, 2002b, p. 9)

My passion for a world in which people attend to each other and respect each other's needs underlies many of my significant choices. The research described in this thesis, which has emerged from some of these choices reflects my passion.

Despite what I wrote about peace, I am not sure that it is the mathematician in me that drives my interest in the infinite. Perhaps it is my interest in the infinite and the impossible that prompts me to explore mathematics, a domain in which we can talk about the infinite and find new possibilities for considering the impossible. Such paradoxes appeal to me.

I am becoming more aware of paradoxes in language as well as in mathematics. With these paradoxes, I am finding more connections between mathematics and language. For example, I have noticed potential for metadiscourse as a way of making connections between referential problems in mathematics and similar problems in language (Wagner, 2003b). Duval's (1999) recognition that "[t]here is no direct access to mathematical objects but only to their representations" (p. 24) could also be said of metadiscourse; discourse about language has an equally problematic set of referents. Structuralist and especially post-structuralist discourses point to the problematic nature of language. No words, whether mathematical or not, have a direct relationship with real-world objects.

Again, I find myself fascinated with a set of phenomena which I was not immediately aware were connected with peace building, my avowed vocation. As with my interest in classroom tasks that direct students to turn their attention to each other in the context of their mathematics, I have recognized the potential of discourse analysis for drawing attention to discourse itself, and to the way people relate to each other in their discourse.

I think it is for this reason that I am particularly interested in critical discourse analysis (CDA), which provides tools for the critical examination of culture. CDA is a tradition within applied linguistics, a tradition in which language features are identified as a way of scrutinizing cultural practices and positioning. CDA is described in greater detail in Chapter 2. When I think of CDA as a vehicle for stepping outside my language practice to examine the practice itself, I see its potential for making the familiar seem strange, its potential for promoting the kind of self-criticism that I have been drawn to as a result of my international experiences.

In this vein, I am more interested in how CDA can be a part of reflective practice than in its substantial potential for casting judgment on others' practice within a discourse and for developing power within the discourse. I am interested in developing understanding, not control or evaluation. In a peer-editing exercise in a graduate course on genres of academic discourse, a peer noted his newly developed discourse analytic skills: "I always have something to say." For me, and I think for him, it is important to say things of value, things that promote reflection, rather than to say things just because we can.

By way of illustration, I find myself critical of others who point away from themselves in their critical explorations. For example, I have noted how Brown (2001) problematized language while failing to notice the same problems in his own writing (Wagner, 2003a). I am aware of the hypocrisy of my criticism. I too am using language as if readers will see my meaning as I intend for it to be seen, and I was using language this way in my criticism of Brown.

I believe that it will be more healthy and helpful for me to examine the potential in language awareness for developing understanding. Though it is far easier and quicker to be judgmental of others, I want to avoid this approach. In my review of Brown's work

I was critical, and in my review and use of significant literature in this dissertation, I will be critical. Though I recognize that criticism is an accepted part of academic writing (see Hyland, 2000), I want to see my criticism as a form of self-criticism, and I hope this way of seeing is evident in my writing.

Conversing with Students

In the research that I will describe in the following chapters, I report on my observations about discourse in the mathematics classroom. Though my conversations were with the students and teacher from one particular mathematics class, I was and continue to be oriented toward mathematics classrooms in general. I noticed that throughout my interactions with these particular people, I strove to see the general in the particular, which is a common practice for mathematicians (see Mason and Pimm, 1984).

Before outlining the flow of this report, I will introduce briefly the people and context of the conversation that is the basis for the report. Chapter 3 provides a more detailed introduction. I participated in a Pure Mathematics 20 class[3] in which the teacher and students were committed to discussing with me and with each other the discourse in their mathematics learning. Because I intended to work with my participants as true collaborators, significant aspects of the nature of my involvement and the involvement of the various participants were negotiated over the course of our conversations. Most of this negotiation was tacit.

I spent a nineteen-week semester with this grade 11 class (although some of the students were deemed grade 12 students), co-teaching the course with the regular teacher and collecting video and audio records of classroom discourse. By directing the students' attention to their own utterances, I tried daily to engage the students in discussion about our language practices in the class. The form of my prompts varied, as I was continually responding to the participants. In addition to our classroom interaction about language, I interviewed participant students and asked them to write accounts of their experiences with language in relation to their mathematics learning.

The data for this study are primarily our conversations about language in class and in interviews, both of which were recorded. I made video and audio recordings of daily class interaction, but I consider most of this substantial collection of data to be secondary to the records of conversation about language. The purpose of this more general data collection was to gather prompts for conversations with the collaborating students and teacher.

Early in my conversations with the students in this study, I shared with them the orientation that I wanted to guide our interactions about our language practice. I told them and wrote out for them the goals I wanted us to share in our critical discussions. In this time of orientation, I was referring to the mathematics class as a game because their class, like other classes and like most games, had its own set of rules and ways of succeeding. This account of my intended orientation is a simplified and abstracted version of the background with which I began this dissertation. Figure 1-1 is a copy of the overhead I showed the students at the outset of our explicit discussion about language practice in their classroom.

When we pay attention to

- what we say, and
- what people around us say

we notice things that we would not notice otherwise.

What can we do with these observations? (these things we notice?)

1. Point fingers (blame)
   saying this is no good, the way things are being done

2. Play the game better (see how to succeed)
   adjusting our participation to fit the system (game) better

3. Consider alternatives
   - *thinking about alternative ways of participating in the system (game)*
   - *trying out these alternatives*

Focus on #2 and #3
Avoid #1

Figure 1-1. Orientation for critical language awareness

Outline of Thesis

As I give accounts of particular streams of our conversation, more details about the participants in this research will be revealed. Some students will be described in greater detail than others, not because they were more important, but because they were key figures in the streams of conversation that captured my attention and because they were more vocal. Some students were relatively silent, and some were completely silent. Silence counts too. Indeed, silence was an important theme in the conversation with students and became even more important in my subsequent interpretations of this conversation.

This chapter's account of my orientation describes only part of the context from which I entered this conversation. My familiarity with scholarship in the fields of mathematics education and linguistics forms another significant part of this context. In Chapter 2, I give brief descriptions of scholarly work that I have consciously considered in my interpretations of my conversations with students and their teacher.

In addition to the published scholarship described in Chapter 2, the most formative background that I brought from the scholarly community into the researched classroom has come from face-to-face communications. For example, at an international mathematics education conference in 2002, Candia Morgan expressed great interest in my desire to explore critical language awareness in a mathematics classroom. She confided that many people, in response to her call for this agenda, had told her that critical language awareness cannot or ought not to be done in a mathematics classroom. I will address their doubts directly in Chapter 9.

At the same conference, I also met Tim Rowland. (Both his and Morgan's work are detailed in Chapter 2.) Like Morgan, he has used discourse analysis extensively to study mathematics learning. I asked him how he would begin a conversation about language practice with the students in a high school mathematics class. He said that to start he would just give them a page from a mathematics textbook and ask them what they make of it. Chapter 5 tells what happened when I did this in my researched classroom.

Chapter 3 describes other aspects of background to the on-going conversation that comprises the data from this research. Three methodological traditions directed my choices in setting up the research. The participant students and teacher and the development of my relationships with them will also be detailed.

Chapter 4 describes the development of my attempts to introduce language awareness in the researched classroom. In this chapter, I will characterize the participant students' response to my language awareness prompts as "passive resistance." While the students usually resisted conversations about language by remaining silent, there were exceptions. Chapters 5 through 7 are accounts of three streams of conversation. Each of these exceptions to the students' passive norm provides insight into the silenced human in mathematics classroom discourse.

In Chapter 8, I look back across Chapters 4 through 7 to consider possible ways for a teacher to respond to mathematics classroom silences. Two short chapters follow. In Chapter 9, I address the warning about critical language awareness in mathematics classrooms that Morgan articulated on behalf of others – "It can't be done." Chapter 10 addresses the potential generativity of silence.

Chapter 2 – Language Counts in Mathematics

> We all paid our personal price for acquiring our language. [...] We all have mastered some aspects of mathematics – what do we truthfully know of the processes and the psychic costs involved? (Pimm, 1993, p. 38)

Language counts in mathematics education discourse because we have no direct access to the objects of mathematics; we can only access the language we use to point at these abstract objects. There has been growing scholarly interest in the language practices of this discourse. Some mathematics educators have adopted approaches from linguistics to study the form of language practice in mathematics classrooms – Candia Morgan (e.g. 1998) and Tim Rowland (e.g. 2000), for example. Others have focused on the nature of mathematics language practice with a more theoretical approach to the signs, symbols and meanings in classroom communication – Raymond Duval (e.g. 1999) and Luis Radford (e.g. 2002), for example. Still others have focused on the social aspects of discourse – Valerie Walkerdine (e.g. 1988) and Paul Cobb (e.g. Cobb and Bauersfeld, 1995), for example.

This chapter represents part of the context of my study of mathematics classroom discourse. My prior reading comprises part of the experience I brought to the research situation. It is impossible for me to represent the sense of this literature that I had before I engaged in conversation with the students and teacher in the researched classroom because experiences in the researched classroom colour my interpretations of any of this work. For example, my perspective on personal pronoun use in the mathematics classroom will be forever changed by my conversations with the research participants.

Furthermore, my understanding of this literature continues to evolve as I engage in the second and third interpretive stages of the research – reflecting and writing.

The chapter also provides the reader with background that will colour and assist in the interpretation of what follows. For this reason, I include accounts of Pickering (1995) and Adler (2001), though before I entered the researched classroom I had not expected their work to be significant in this research. The emerging significance of these works highlights the complex nature of time in thesis writing. There are a number of temporal settings, including the research experience itself, informal interpretation in which I wrote notes for no particular audience, and the writing of this dissertation, which was another stage of interpretation. The work of Adler and Pickering did not figure prominently in the first stage, but grew in importance in the second and third stages.

This review of scholarship addresses the question: why did I choose this kind of study? In this sense, it is closely related to methodology. Method is made up of choices. First, I describe my interest in discourse analysis, and consider mathematics education scholarship in this area. Second, I consider alternatives that I did not choose. Third, I differentiate between critical and descriptive approaches to discourse within the framework of an interest in lexico-grammatical features. Having chosen a critical approach, I describe some methodological implications of this choice. This chapter does not give an account of my overall interest in mathematics classroom discourse because this appeared in the first chapter. Nor does this chapter address my choice of a methodological framework, because that will appear in the third chapter. However, a significant overlap remains between the third part of this chapter and the methodology choices described in Chapter 3.

Studies of Lexico-Grammatical Features of Language

There are many ways in which discourse can be studied. For this particular study, I chose to direct my attention to lexico-grammatical features of classroom language form. Halliday (1978), who is one of the first modern scholars to write about language and mathematics, includes words and linguistic structures in the category of *lexico-grammar*. Though words and structure were the focus I had in mind when I began the research, the conversations with participant students drifted to other aspects of language. For example, the stream of conversation in Chapter 7 relates closely to semiotics.

My interest in lexico-grammatical features of language emerged from my interactions with David Pimm, who shares this interest. Early in my graduate studies I was captivated by his article “The silence of the body” (Pimm, 1993). This piece heightened my awareness of the significant relationship between words, culture and the place of the person in a discourse. Pimm does not directly address his chosen title in this poetic article. The reader is left to wonder what is silent about the human body in mathematics and what the body of mathematics is silent about. In retrospect, I see the article as being seminal in my research, as I became increasingly aware of the silenced human in the mathematics classroom.

Voice

In my conversation with students in this research, issues relating to voice quickly became important. In grammar, *voice* refers to the position of the subject relative to a verb. The *active voice* has the subject acting with agency, as in “*I* hit the ball.” The *passive voice* has the subject receiving the action, as in “The ball was hit by *me*.” The passive voice can also be used to obscure who has agency, as in “The ball was hit.” This utterance does not necessarily imply that there is no agency, but it hides who in particular has the agency – it could be anyone or anything that hit the ball. Thus, voice has to do with agency.

The word *voice* can also refer to the level of input a person has in a discourse. For example, in Carter's (1993) argument for the importance of listening to teachers, she writes,

> [T]he issue of voice centres on the extent to which the languages of research on teaching, with their emphasis on general propositions, allow for the authentic expression of teachers' experiences and concerns. [...] the issue is one of discourse and power, that is, the extent to which the languages of researchers not only deny teachers the right to speak for and about teaching but also form part of a larger network of power. (p. 8).

I am saying in this dissertation that we need to draw out the voice of students as we investigate mathematics learning discourses. In this sense, *voice* can be an opposite of silence. However, if *voice* refers to a person's level of input, the "input" need not be linguistic. A person can make a contribution with a gesture, with silence or by merely being present.

The word *voice* can also refer to a person's unique character. In this sense it is similar to *face*. We recognize people by their faces even when we cannot explain how we distinguish between them. In the same way, we can recognize a person's discursive style by his or her diction, word-choices, sentence structuring, tone and reasoning. For example, I can sometimes recognize the writing of a person I know well without seeing the person's name attached to her or his work, even if it is printed in a standard font.

Blind reviewers of scholarly articles have noted this phenomenon. They recognize certain scholars' *voices*.

These three senses of the word *voice* are interrelated. When we speak or write with the active voice, as opposed to the passive voice, we are not masking our agency in the situation. If we refer to our agency, we suggest that we are making a contribution to the situation. And when we make contributions, we tend to do it in our own unique ways. We express our voices.

Chapter 5 is an account of a conversation about voice. In this stream of the on-going conversation, I never defined the word *voice* for the students. Rather, I began by directing their attention to agency. Soon after, I began talking about an *I* voice, a *you* voice and a *we* voice. By implication, this use of the word *voice* included a sense of active voice, because of our initial discussion about agency and a sense of the person having a voice – in the sense that the person contributes to the conversation in a unique, personal way.

In my interpretation of the stream of conversation about agency, I drew on Pickering (1995), who distinguishes among three kinds of agency – human agency, disciplinary agency and material agency. What he calls *human agency* refers to the conventional meaning of agency as expressed in active voice. When I say, "I hit the ball," it is I, a human, who am doing something. By contrast, "I am stuck" conventionally suggests that there is no agency. Pickering would call this an expression of material agency, because it is the mud that has me in its grasp. The mud, which is concrete material, is doing something. And sentences that begin "The survey shows the necessity of …" exemplify what Pickering would call the expression of disciplinary agency. Here the influential factor is a discipline that the people in a community follow.

Bakhtin (1953/1986) also distinguishes between different agents in linguistic expression:

> any word exists for the speaker in three aspects: as a neutral word of a language, belonging to nobody; as an *other's* word, which belongs to another person and is filled with echoes of the other's utterance; and, finally, as *my* word, for, since I am dealing with it in a particular situation, with a particular speech plan, it is already imbued with my expression. (p. 88)

He points out a tension that he asserts is present in all communication. The tension is between the first aspect of meaning – the neutral word – and the other two expressive aspects of meaning – my word or the other's word. There is on the one hand personal, temporal meaning, and on the other hand meaning that belongs to no one. The meaning that belongs to no one is often seen to be the meaning that everyone accepts – dictionary definitions, for example.[4]

Bakhtin's recognition of the tension between expressive and neutral meaning highlights the complexity of analyzing voice in any utterance. It is relatively simple to identify grammatically whether a sentence is active or passive in voice. However, it is not so simple to identify meaning. When a teacher utters the active-voiced sentence "We complete the square," she seems to be suggesting that this process is required by the discipline she represents. Thus the discipline has agency and she likely intends the words in her utterance to have their conventional meaning. The analysis of "I complete the square" is much more difficult. A person saying this may be contributing an innovation in an investigation, and therefore be acting as a human agent. Alternatively, this person may be referring to a conventional approach, as in "I am to complete the square." In either of these utterances, a particular word may have special meaning in its context and thus have meaning as an expressive word in addition to its meaning as a neutral word. It is difficult to distinguish between the voice of an individual and the voice of a discipline for which the person could be a representative.

A few mathematics education scholars have investigated voice. Pimm was a forerunner of the study of language features in mathematics learning discourse. In his exploration of language issues inherent in mathematics teaching and learning (Pimm, 1987), he looks at pronoun use, among other things. He is interested in the frequent use of the pronoun *we* in mathematics classroom language practice. He asks, "Who is *we*?" after noting that students and teachers use the word *we* extensively and with various apparent referents. He suggests that a person's use of the pronoun *we* can be a clue to expected cooperation, assumed complicity, or generality. Indeed, it makes sense for a collective to have agency, to have influence on a situation, but the pronoun *we*, which refers to a collective, is typically vague about who is comprised in the collective that is being suggested.

Tim Rowland, who was a student of Pimm's, took up an interest in such vagueness. He extends Pimm's investigation of pronouns with further qualitative inquiry. He is captivated by the ambiguity of the word *it* in mathematics dialogue (Rowland, 1992, 2000). He also extends Pimm's analysis of personal pronouns and investigates the use of *I*, *you* and *we*, in addition to further considering of the pronoun *it*. He draws on Mühlhäusler and Harré (1990) in his discussion of the way mathematics teachers and learners use language to point vaguely at context, both with personal pronouns and with other words like *this* and *that*.

Chris Bills (2002) has also investigated children's use of pronouns when conversing about mathematics. His is a largely quantitative study of some clinical interviews with elementary students. He identifies disparity between higher and lower achieving mathematics students in their use of pronouns and verb tenses when they refer to (point at) their mathematical thinking. In his study, the higher achievers did not use the pronoun *I* as often as their peers did.

Deixis

Pronouns are a form of deixis, which is pointing with language. For example, when I say "I", I point at myself with the word. The root of the word *deixis* is the Latin word for *finger*. If we think about deixis broadly, we could say that many words are mere pointers. They point at other things. And all such pointing is inherently vague. There is no one-to-one correspondence between meaning and object. The study of the way words point is called semiotics, which will be discussed later in this chapter.

In pragmatics, which is the field of linguistics that is interested in the effects of people's language choices, deixis has further significance. For example, in linguist Stephen Levinson's (1983) major book about pragmatics, there is a chapter on deixis. He distinguishes between distal pointers, such as *that* and *there,* and proximal pointers, such as *this* and *here*. In a reanalysis of transcripts from my master's research, I considered the deixis in student–teacher interactions in two settings: mathematical investigations, and my interviews with the participants in the investigations (Wagner, 2003b). I noted that verb tense could give a sense of proximity or distance, with the present tense indicating proximity and the past tense indicating distance.

Linguist John Lyons (1977) in his major work on semantics, also notes deictic features of verb tense, but his interest is not a pragmatic one. (Semantics refers to the meaning structure of language forms.) For him, tense "grammaticalizes the relationship which holds between the time of the situation that is being described and the temporal zero-point of the deictic context" (p. 678). He describes how the different tenses point to different times. In my reanalysis of transcripts, I was interested in the participants' sense of attachment (or proximity) to mathematics, to the classroom tasks and to their own utterances. Like the tenses, the pointers *this, that*, *here* and *there* also have a literal sense, in that we use them to distinguish between something close by or something farther away. My interest for transcript analysis was in the way these words were used to point at things that are neither close nor far – things like mathematical objects (which do not have concrete presence), a person's idea (whether mathematical or not), or a person's utterance. In such cases, a switch from proximal to distal pointing, for example, can suggest a speaker's intention to distance the thing from herself, and a switch from distal to proximal pointing can suggest a growing sense of attachment to the object of attention.

Deixis will feature prominently in Chapter 6, which is an account of a stream of conversation about the way participants direct their attention in mathematics communication. In any conversation, not only do people direct their own attention, but they also use deixis to direct each other's attention.

Modality

Modality, which refers to the means by which speakers express their attitude about their own propositions, is another lexico-grammatical feature of mathematics learning discourse that has been studied within the pragmatic tradition. Rowland (2000) includes discussion about modality in his exploration of vagueness. Modality can be expressed with modal verbs or with hedges. For example, in the sentence "You can complete the square in this expression," the modal verb *can* expresses the speaker's confidence that his audience is capable of completing the square. Rowland (1995) describes hedges as "words which have the effect of blurring category boundaries…[and which] hedge the commitment of the speaker to that which s/he asserts" (Rowland, 2000, p. 58), words such as *sort of*, *I think*, *maybe*, and *perhaps*. He uses the taxonomy of hedges developed by Prince et al. (1982) in the context of medical practice to classify various language tools that students use to protect themselves from being proven wrong, and points to Lakoff (1972), who introduced the term *hedge* for this kind of self-protection. From this, Rowland highlights the importance of giving students space to be unsure, calling this space the *zone of conjectural neutrality* (Rowland, 1995).

Chapman (2003) has also analyzed mathematics classroom texts in terms of modality. She finds that with relatively little prompting from their teacher, more successful students are able to phrase their mathematics talk with high modality (a high degree of certainty) and using metonymic rather than metaphoric form. She reports that students develop mathematical understanding as they move from less mathematical to more mathematical form in their speaking, from low to high modality, and from metaphor to metonymy. This ability to adopt mathematical textual form she calls *transformational freedom*.

In my conversations with students in this research, we talked about modality a few times, but no students ever seemed to be captivated by these discussions. Their relative silence on this topic was typical of the silences I will describe in Chapter 4. It attests to the importance of the streams of conversation in which students did not remain silent.

Using Lexico-Grammatical Analysis

Rowland's investigations of vagueness differ significantly from other mathematics education scholarship that considers lexico-grammatical features. Rowland studies vagueness, one lexico-grammatical feature of language practice, and draws on various mathematics learning situations to support this investigation. Others have studied more particular situations and have drawn on various lexico-grammatical features to support their investigations.

Morgan (1996; 1998), for example, is interested in students' mathematical writing. In her extensive investigation of mathematical writing, she uses various lexico-grammatical features to explore the writing. Her interest seems to shift from mathematics writing in general, to student writing in response to mathematical investigation tasks, and finally to teachers' assessment of this writing. As part of her investigation of students' written texts, she notices thematic and content features as well as language features, including nominalization (making a word into a noun), voice and the use of personal pronouns, imperatives and modality.

Most significant to the formation of my research agenda are her references to genre, her more explicit adoption of critical linguistics in her interpretation of textual features and her call for further inquiry. She spends the bulk of her energy trying to understand a phenomenon in mathematics classrooms. Only in her final thoughts does she direct her attention to asking what might be done for mathematics students. She explicitly calls for the development of critical language awareness in mathematics students, though she does not say much about how this might be done. It is in response to Morgan's plea that I began to think about taking up this call.

Herbel-Eisenmann (2000) also draws on lexico-grammatical features of language practices in her study of two reform-oriented classes that were using textbooks from a new series, the *Connected Mathematics Series* (e.g. Lappan et al., 1998). In addition to examining the content of classroom discourse, she looks at features of language form in both oral and written classroom texts to identify classroom norms. In her call for further research, Herbel-Eisenmann, like Morgan (1998), notices the voiceless participants in classroom discourse, but does not say how they could be brought into the conversation: "I am again continually aware of the fact that student perceptions have once again been ignored. [...] How do students make sense of the discourse patterns in the classroom?" (p. 284). She encouraged my desire to take up Morgan's call for critical language awareness, to begin thinking about bringing students into the conversation about language and mathematics, a conversation that until now has involved mostly researchers and some teachers.

In her analysis of the textbook *Thinking with Mathematical Models* (Lappan et al., 1998) Herbel-Eisenmann (2000) counts questions and imperatives, and categorizes them according to Rotman's (1988) schema, distinguishing between imperatives that tell a reader to be a "thinker" (e.g. *explain*) and imperatives that prompt the reader to be a mere "scribbler" (e.g. *draw*). While Herbel-Eisenmann laments the proportional absence of thinker imperatives, Rotman, in his development of a semiotics of mathematics, recognizes that mathematicians scribble *and* think. Herbel-Eisenmann also looks at the use of personal pronouns, modality and hedging.

Herbel-Eisenmann's chapter in which she investigates the textbook is inspired by Pimm, who, with his colleague Eric Love, suggests that textbooks should not be taken for granted: "[teachers' and students'] responses to [their textbook] may range from taking it for granted to seeing their role as challenging and criticizing it (to interrogate and even deconstruct the text)" (Love and Pimm, 1996, p. 380).

Phillips (2002) has examined mathematics textbooks with her students. She had her Grade 4 students write a mathematics textbook for their successors. Her students adopted some of the features of the textbooks to which they were accustomed but their writing also had significant differences. She locates the motivation of her inquiry in her growing disillusionment with mathematics journal writing, a practice that was being promoted in the early 1990s. Here is Phillips' account of the result of asking her students what they thought about journal writing in mathematics:

> I asked these pupils about my suspicion of their using a 'formula' for writing in mathematics classes, and they agreed, opening the door for a torrent of complaints. They told me that they hated writing in mathematics and could see no purpose for it. Some even said that writing was making them dislike mathematics, because one of the things that mathematics traditionally offered was "not having to do all that written stuff". (p. 3)

With their affirmation of her suspicion, she set out to try other forms of writing in which both form and content could be addressed simultaneously.

Phillips became increasingly interested in five perspectives for looking at texts: audience, purpose, form, content and voice. She talked with her students about these five perspectives, though she used different words in her discussion with them:

> Currently, when my pupils write in mathematics class, I ask them to consider their answers to five related questions as they start to write. *Who* am I writing for? *Why* am I writing this? *How* do I expect the finished product to look? *What* do I know and/or what am I thinking about this topic? *Where* am I in relation to the writing? (p. 13, emphasis hers)

The *who* question relates to audience, *why* to purpose, *how* to form, *what* to content and *where* to voice.

It seems to me that Phillips has found one effective way to address Morgan's call for critical language awareness – engaging students in writing mathematics and about mathematics using various genres, and talking with them about their and other authors' texts.

Other Forms of Discourse Analysis

In addition to the scholarship described above, there are other forms of discourse analysis, forms that do not focus attention on lexico-grammatical features. Mathematics educators have also drawn on some of these forms of investigation. In the following paragraphs, I will make connections between this scholarship and my research.

Genre Analysis

In a review of Morgan's (1998) book, Pimm and Wagner (2003) found important both the clarity with which Morgan showed how students were being encultured to write in particular forms, and her thoughtfulness about the appropriacy of explicitly teaching students to write in these forms. She mentions the word *genre* periodically, but does not seem to consider the possibility that student write-ups of mathematics investigations could be seen to form a unique genre. The review identifies the nature of genre as needing more work in the context of mathematics classroom discourse.

Bakhtin (1953/1986) points at the complexity and diversity of speech and recognizes that a genre can be large enough in scope to encompass an aspect of a global discourse system, or as small as a short conversation between two people. No matter what guidelines are used to distinguish a type of writing or speaking as a genre, the linguistics tradition of genre analysis can be applied to mathematics texts. Typically, the purpose of promoting awareness of genre features seems to be to empower people to write or speak within the genre being studied. For example, Hyland's (2000) analysis of genres of academic writing includes a chapter on teaching these genres and a chapter in which he gives rationale for the imperative.

However, not all genre analysis has this intent. Gerofsky's (1996, 2004) analysis of mathematical word problems, for instance, does not seem to be aimed at helping educators write word problems well; though in her book (Gerofsky, 2004), she does relate accounts of how some educators come up with their word problems. Rather, she seems to be interested in helping educators *understand* the genre so that word problems can be used in a healthier manner. She closes a journal article (Gerofsky, 1996) with this hope:

> I do feel that it is important to think in new ways about the nature and purposes of word problems, about their inherent oddness and contradictions, and about our rationale for using them in school mathematics programs, rather than simply, unthinkingly visiting them upon future generations of schoolchildren. (p. 43)

She seems to be using genre analysis for critical reflection rather than for empowering people to write well within a particular genre. It seems to me that her interest is in helping people live well, not in helping them perform well.

By contrast, Solomon and O'Neill (1998), in their genre analysis, seem to be more interested in helping students perform well when they identify particular features of mathematical writing. As with Gerofsky's agenda, their approach can be connected to critical, emancipatory discourses. Increased attention to genres in school can help balance the distribution of linguistic-cultural capital (see also Martin, 1989; Halliday and Martin, 1993; Gilbert, 1994). Solomon and O'Neill attribute their interest in identifying genre characteristics to Martin, Christie and Rothery (1994), who argue "for the explicit teaching of the specific features of written genres, since these cannot simply be worked out by children in the course of reading and writing activities or from teachers' clues" (Solomon and O'Neill, 1998, p. 211). Solomon and O'Neill argue against Burton's (1996) promotion of narrative form in students' mathematical writing. To support their argument, they identify features of the writing she promotes from her own examples and compare them to features of some mathematicians' correspondence.

While I find their comparisons fascinating and useful in my developing awareness of language forms in mathematical writing, I am disturbed by their assumption that students ought to be taught explicitly to write the way a mathematician writes. There are significant differences between the contexts in which students and mathematicians do their mathematics. This problem – Solomon and O'Neill's (1998) assumption that all mathematical writing should share common features – could be a result of a too narrow conception of genre. Indeed, there are many genres in mathematics and many in mathematics classrooms.

I have two concerns with their conclusion. First, I ask which genres in particular are necessary or helpful for students to learn, if the explicit teaching of genres is helpful at all. There may be value in inducting students into the genre they promote, but there may also be value in asking them to write within different genres of mathematical writing. This concern might be alleviated with the teaching of genres accompanied by discussions with students about the connections between the genres and the content, intents and values with which they are associated.

The second concern I have with Solomon and O'Neill's (1998) imperative is the loss of meaning that seems to accompany the teaching of form in the absence of content. The tension between attending to form or to content is also addressed by Jill Adler. In her investigation of issues relating to teaching mathematics in multi-lingual environments, she identifies three dilemmas that exist for all teachers but that are exaggerated in a multi-lingual context (Adler, 2001). The dilemma between implicit and explicit language practices, which she calls the *dilemma of transparency*, became increasingly important in my research context and in my interpretations of the research interactions. This dilemma will be considered in greater detail in Chapters 5 and 6.

My conversation with students in this research relates to this discussion about the explicit teaching of genre, or in Adler's words, the explicit teaching of language. In my interactions with participant students, we attended to excerpts of our oral mathematical interaction, which could be considered a genre in itself. We identified features of the genre and investigated possible meanings and alternatives. While I recognize the importance of fluency in mathematics (the ability to use language as though it is transparent,) I suggest that there is value in attention to language.

Halliday (1978) also addresses this dilemma:

> [T]here is a feeling shared by many teachers, and others concerned with education, that learning ought to be made less dependent on language; and teachers of mathematics, in particular, emphasise the importance of learning through concrete operations on objects. This is a very positive move. At the same time, there is no point in trying to eliminate language from the learning process altogether. Rather than engage in any such vain attempt, we should seek equally positive ways of advancing those aspects of the learning process which are, essentially, linguistic. (p. 203)

Conversation Analysis

Richard Barwell shares Adler's interest in multi-lingual mathematics classrooms. In his dissertation (Barwell, 2002), he expresses a concern about the inaccessibility of meaning and asserts that for him this problem is exacerbated because he comes to his research from an extra-cultural perspective. With this problem in mind, he chose to research patterns of attention in students' mathematical conversations. This approach to interpreting language practice is called *conversation analysis*. Barwell describes the benefits of this approach that expresses an interest in the meaning ascribed to utterances rather than to the intentions the speaker might have: "[T]he analyst can avoid attempting to say what participants intended to do by their words, by looking instead at how those words are treated by the other participants" (pp. 41-42).

In Levinson's (1983) comparison between discourse analysis and conversation analysis, he describes the latter as follows: "[T]he emphasis is on what can actually be found to occur, not on what one would guess would be odd (or acceptable) if it were to do so" (p. 287). My research is not a study of what actually occurs. It is not a descriptive study. It is a study of possibility, a critical study. Conversation analysis does not seem to fit the critical questions I have in mind.

Though conversation analysis and discourse analysis are seen by linguists to be mutually exclusive, I include conversation analysis in my list of other approaches to discourse analysis because it is a form of analyzing discourse. It is an approach to discourse that I might have taken.

Semiotics

Another approach to discourse is to explain the relationships between words and the things to which the words refer. This stream of scholarship, which was introduced by Saussure (1959), is particularly interesting in mathematics because of the unique role of symbols and the abstract nature of the objects of study in its discourses. Presently, there is considerable scholarly interest in signs and symbols as they relate to mathematics learning. Pimm was a forerunner in this scholarship with his book *Symbols and Meanings in School Mathematics* (1995), in which he explores the connections between representation, manipulation, experience and meaning in the context of mathematics learning.

A semiotics discussion group has met annually since 2001 at conferences of the International Group of the Psychology of Mathematics Education. Duval (2001), in a paper presented in the first gathering of this series, underscores the critical nature of semiotics in the context of mathematics learning:

> Because mathematical objects, in contrast to phenomena of astronomy, physics, chemistry, biology, etc., are never accessible by perception or by instruments (microscopes, telescopes, measurement apparatus,…) *Access to mathematical objects must necessarily pass by way of semiotic representations*. [...] One can therefore formulate the paradox of comprehension in mathematics in the following way: *how can one not confuse an object and its representation if one has no access to this object apart from its representation*? (p. 7, emphasis his)

In the same discussion group, Sáenz-Ludlow (2001) notes a tendency in mathematics classrooms to believe that meaning rests on symbols, independent of context. She describes the importance of semiotic inquiry this way: "Understanding the semiotic aspects of classroom discourse will elucidate the relationship among students' mathematical activity, teachers' mathematical activity, and the interactions between teacher and students and among students themselves" (p. 6-7). I agree with her imperative for developing an understanding of semiotics. However, I wonder who she has in mind when she speaks of improving understanding. I think she intends for researchers to understand better each of these kinds of interactions, but I wonder what is the value of such understanding if it is not shared with classroom participants – teachers and students.

Indeed, this question underpins my research agenda. In Chapter 6, I give an account of students in my research showing increasing understanding of the semiotic relationship described by Sáenz-Ludlow.

The importance of semiotics to mathematics education comes down to the problem of using symbols to refer to things that have no existence outside the imagination of people. Otte (2001) puts it this way: “Within mathematics there is no absolutely fundamental ontological level” (p. 2). In mathematical discourse we use words to point at things that simply do not exist in such a way that we can look at them together. In geometry, for example, I may draw a diagram to represent what I imagine, but even a diagram cannot capture my image (cf. Tahta, 1989). Diagrams are static, and may point to different things. This problem is central to my conversations with two students in this research. This conversation will be detailed in Chapter 6.

Pimm (1995) illustrates the nature of representation by considering a triangle. Making reference to Magritte’s painting “Ceci n’est pas une pipe,” he shows a diagram of a triangle and asks if it makes sense to assert that it is not a triangle, but that it is merely a drawing of a triangle. He notes the typical use of metonymy to refer to the real objects of mathematics that are inaccessible. It is significantly different talking about a pipe, which my conversation partners and I can all experience, than talking about a triangle, a parabola or a proof, which only exist somehow in our imaginations. In Chapter 6, I give an account of my attempts to direct participant students’ attention to semiotics. In this discussion I used the Magritte painting and an image of a square.

In addition to addressing questions about representation in general, semiotics scholarship relates to my interest in silence. Radford (2002) calls pointing at something that is not there "deixis at phantasma." He notes how in mathematics we can point to something not there, and so make it evident to another person. For example, I can talk about the intersection of the diagonals in a square without actually drawing a square, its diagonals or the point of intersection. Anyone in my audience who has an understanding of the conventional meanings of my words ought to be able to "see" the square, the diagonals and the intersection point without looking at a diagram. Radford calls this *objectifying deictics*. However, the student's problem that Duval raises still remains. Deixis at phantasma can become objectifying deictics only when the audience is able to conjure up its own representation of the thing that is not there. Surely, such emergence of what Radford calls a "whatness" is beyond the control of anyone in a conversation.

Rotman's (1987) account of the introduction of the mathematical sign zero into Western consciousness can be seen as a metaphor for the wider semiotic problem in mathematics. He entitles his book *Signifying Nothing: the Semiotics of Zero*. In a sense all mathematical symbols signify nothing. At least they signify something that does not exist, which is not the same thing as signifying nothing. With the closing chapters of this dissertation, I will ask questions about representing and researching *silence*, a particular kind of nothingness.

Socio-cultural Analysis

There is a range of mathematics education scholarship that directs attention to socio-cultural issues, such as the relationship between discursive practice and discursive systems. Rotman (1987) calls his historical account an *archaeology* in the Foucauldian sense. Foucault distinguishes between discursive practices and discursive systems. *Discursive practice* refers to the actual utterances in a situation, and *discursive system* refers to the language and relational structures that allow particular utterances to have meaning. Meaning arises from the system within which an utterance is made as much as it arises from the utterance itself.

Walkerdine (1988) illustrates the contextual nature of meaning in her extensive study of comparative words used in English language homes and in English language classrooms. She argues that the relationship between words and their intended meanings – that is, between a signifier and a signified – is always problematic: "[R]eference is not a universal, but rather an aspect of the regulation of social practices which form the daily life of young children" (p. 11). For her, not only are words signs (in relation to their referents), but also the student becomes a sign because of the educator's efforts to make him "normal." When teachers direct their students' understandings toward particular predetermined outcomes, whether these outcomes are explicitly acknowledged or unconscious (these are the more powerful ones), the teachers as subjects operate on their objects, their students.

While I share Walkerdine's concerns about the "normalization" that occurs in mathematics classrooms, I recognize its inescapability. I operate as a subject in any action I take in any sort of relation. When I do this, I make a difference for the people with whom I relate, and I objectify them to some extent. At the same time, I become objectified by them. The objectifying nature of relationships is more important in classrooms because of the unequal power relations that exist there, and so I believe that teachers need to be cautious. Walkerdine worries most about unrecognized normalizations. Indeed, I believe that the value of her work is that it brings to awareness a problem inherent in educational practice – the power of language in the normalization of children.

Paul Cobb meets head on the question about normalization and considers the nature of desirable norms in mathematics classrooms. He has written extensively and with numerous scholars about the nature of good classroom discourse (e.g. Cobb and Bauersfeld, 1995). While this stream of his work asks how the discourse *ought* to be and how a teacher can direct a classroom to such a desirable practice, other scholarship (some of which I have described above) describes how the discourse actually *is*. My research is different still. I am interested in how the discourse *might* be and how awareness of these possibilities affects how it *is experienced*.

Cobb also contributes to a largely American interest in the overlapping and clashing of discourses. This stream of scholarship is well-represented in a special issue of the journal *Mathematical Thinking and Learning* (Cobb and Nasir, 2002). Here Cobb writes the word *discourse* with an upper case *D*, and particular "Discourses" are equated with particular cultures (Cobb and Hodge, 2002). While I consider this scholarship to be significant, I find myself frustrated with its relatively narrow view of culture and discourse. It seems that in this research all cultural issues relate to an imagined line drawn between Blacks and Whites or between Hispanics and mainstream English-speaking whites. It seems to me that much of the analysis could be extended to describe more subtle cultural differences in addition to the glaring and truly problematic cultural disparities to which it refers.

Politeness

The final form of discourse analysis that I describe here is referred to as *politeness theory*. It also takes an interest in social interaction, but on a more individual level than the socio-cultural approaches I have just described. Brown and Levinson (1987) introduced the term *politeness* to refer to accounts of social interaction in terms of language forms. Liz Bills (2000) applies their classifications of face-threatening acts (FTAs) to mathematics classroom dialogue. She identifies a wide variety of politeness strategies in some teacher-student interactions and comments that the teacher's "politeness strategies [...] aimed to reduce the appearance of sapiential power [, power that is attributed to unequal knowledge,] by giving weight to the students' opinions, wishes and cognitive concerns" (p. 46). Weingrad (1998) also draws on Brown and Levinson's framework, but introduces more terminology than Bills does. She notes an increasing potential for FTAs in the context of mathematics curriculum reform that encourages increased classroom dialogue (e.g. NCTM, 1991).

I mention this scholarship only to distinguish it from Chapter 6's discussion of the way students face mathematics. In politeness theory, the word *face* figures prominently. Terminology includes *face threatening acts*, *positive face* and *negative face*. The word *face* is used figuratively. In my sixth chapter, I will describe a stream of conversation regarding the way participants turn their faces in mathematics conversation. I will use the word *face* literally – we turn our actual physical faces when we turn our eyes to look at something. Though I am pointing out a difference between literal and figurative usage of this word, I recognize the potential for a generative investigation of the connections between figurative face and the way people position and turn their literal faces.

The Orientation of Discourse Analysis

In the above descriptions of scholarship regarding discourses in mathematics education, I concentrated on the form of the analysis. Another important factor in any analysis is its orientation. What is the purpose of the research? I suggest that all of the mathematics education research that I described is oriented to pedagogy to some extent. It addresses issues significant to teachers, particularly mathematics teachers. However, within this orientation, one can be interested in understanding what exists already or in investigating possibilities for change. I distinguish in this way between descriptive and critical investigations.

Critical versus Descriptive Analysis

To sharpen the distinction between descriptive and critical orientations, I draw upon an on-going conversation between two linguists. Michael Stubbs is a linguist who began describing discursive practices in schools thirty years ago (e.g. Stubbs, 1974). Norman Fairclough's early linguistics research also was descriptive, but he has become known for his interest in the relationship between language and power (e.g. Fairclough, 1989). He calls investigations of language and power *critical discourse analysis* (CDA) (e.g. Fairclough, 1995).

Fairclough and Stubbs have been engaged in an on-going conversation about critical discourse analysis by publishing criticisms of each other's publications. Because they have different goals, their debate is unlikely to be resolved, but it has nevertheless been generative. Their awareness of each other seems to have shaped and strengthened their work. Stubbs (1997) reveals his agenda when he suggests improvements to CDA that would sharpen its capacity to develop a "systematic and thoroughly documented study of cultural transmission" (p. 114). Fairclough does not seem interested in developing this kind of study. Instead, he is interested in investigation that identifies "the range of what people can do in given structural conditions" (Chouliaraki and Fairclough, 1999, p. 65). Stubbs is also interested in the variation within discourses, not as a way of exploring possibilities for living within them, but rather because thorough description would require the consideration of variation as a means for discourse participants to convey meaning:

> All language in use shows variation, and it may simply be that the authors were trying (consciously or not) to vary their grammatical choices for stylistic reasons. However, if these choices tend in a particular direction, then these tendencies may nevertheless convey meanings to readers. (Stubbs, 1996, p. 126)

Stubbs is interested in thorough description and Fairclough is interested in exploring possibilities. One wants to say how a discourse *is*, and the other wants to say how it *could be*. Despite their differences, both Fairclough and Stubbs suggest an ethnographic component and demand the consideration of a large body of textual material, either a corpus comprised of transcripts of oral discourse or samples of written discourse.

To illustrate the connection between large corpora and ethnographic considerations, I draw on Sinclair (1965), who notes the importance of large corpora:

> Any stretch of language has meaning only as a sample of an enormously large body of text; it represents the results of a complicated selection process, and each selection has meaning by virtue of all the other selections which might have been made, but have been rejected. (pp. 76-77)

Though the range of possibilities is very large in a discourse, it is far narrower for any particular participant in any particular moment. Someone reading or hearing an utterance finds meaning by comparing it to her set of language experiences and taking it in relation to other language she sees as being in a similar genre. To have access to the meaning ascribed to utterances, we must be members of the discourse's community, or at least be sufficiently immersed in it. This suggests the need for an ethnographic component to the research.

Methodological Implications of a Critical Orientation

For more careful analysis, I take two recent major studies published by Falmer Press in its "Studies in Mathematics Education Series." I described both studies earlier in this chapter. Now I will direct my attention to different features of their work. Instead of focusing on the content, I will focus on their orientation and their methodological choices. I will evaluate these two studies in terms of Fairclough's and Stubbs' differing and common concerns. I will ask the following questions of these studies: Is it descriptive (following Stubbs' paradigm) or is it critical (Fairclough's paradigm)? Did the researcher consider a large corpus of utterances? And, was there an ethnographic component to the researcher's investigation? I ask these questions of Rowland's descriptive work and Morgan's critical work to garner methodological implications for my research, which shares Morgan's critical orientation.

In Rowland's (2000) examination of vagueness, he examines transcripts from a substantial collection of clinical interviews to see which grammatical constructions students used to express vagueness and certainty. In his words, his aim is "to access and describe the mathematical frameworks and private constructions locked away in children's minds. [...] to uncover what they 'knew' and how they structured that knowledge" (p. 1).

Though he seems to describe an interest in individual students' private constructions, his conclusions suggest that he is interested in students in general, in the connections between their public, spoken mathematics and their private constructions. In any education research there is this tension, the sense of compulsion to generalize from investigations of particular moments in particular students' lives. Within this tension, there is a sense that studies drawing on relatively large corpora of language, such as Rowland's study, contain more valid generalizations. We need to be careful. This standard for validity assumes that the discourse is homogeneous. We can expect the language practice of individuals in large or small discourses to tend toward uniformity, but it is often a surface uniformity. Consider, for example, the different sense women and men have had of gender-exclusive language.

As educators, we may have a sense that we see the same thing as each other when we look at particular artefacts of mathematics communication, but this sense probably depends on the questionable assumption that we share relatively common experiences of mathematics classrooms. At a recent conference, I participated in a working group that analyzed videos and transcripts from classrooms in South Africa and Pakistan (Barwell, Halai and Setati, 2003). It seemed to me that our discussion demonstrated both our tendency to assume that we all shared the same interpretations, and the fallacy of this assumption. However, there is value in making and discussing generalizations, despite their inherent flaws.

Rowland's analysis ostensibly describes the nature of certain language practices in school mathematics, but the utterances that he evaluates come from clinical interviews. This is a significant distinction because of the prevalence of clinical interviews in the research of mathematics learning. For my research, I needed to choose between clinical interview discourse and classroom discourse as my primary source of language practice. Though clinical interviews have advantages, their contexts differ significantly from that of the classroom. Indeed, they differ from the significantly wide range of cultures in the many mathematics classrooms throughout the world. Participant children's clinical interview utterances can be expected to differ from what they would say in classrooms, both in content and in intention.

Both Fairclough and Stubbs assert the need for an ethnographic aspect in investigations of language practice. It is not clear to me what particular culture a researcher should immerse herself in to interpret adequately the discursive practice in a single clinical interview or set of interviews. The interview itself has an ephemeral culture. The researcher and each participant bring elements of their cultures to the situation. Together they form a new mode of interaction, which disappears as soon as the interview ends. A series of interviews may appear to have the same culture because the researcher has the principal formatting power in this process, because participants tacitly agree or even sign waivers of consent to submit to the interviewer's structuring, or because sets of participants often come from similar situations. However, in typical interview series, the researcher is the only person who moves freely from one fleeting interview culture to another.

I suggest that the misleading nature of interview cultures promotes our desire to translate apparently homogeneous interview interactions into classroom cultures to make statements about mathematics teaching and learning. With this translation problem in mind, to include an ethnographic component in the analysis of clinical interview utterances seems nonsensical, if not impossible. Because of the fleeting nature of the interview culture, the ethnographic component must be displaced to a culture other than the one in which the analyzed utterances are made.

Interview participants carry with them numerous sets of cultural assumptions. Are their mathematics classroom cultures the only ones of significance, or are there others? Barwell (2002) addresses this question and concludes that he need not have visited his participant students' homes. He considered his immersion in their mathematics class sufficiently informative for his interpretation of their small-group mathematical interactions.

Though the complex nature of cultures can challenge any ethnography, it is necessary to find ways of including an ethnographic component in discourse analysis because of the contextual nature of meaning. The meaning of any particular utterance is as dependent on the words as it is on their context. While Stubbs (1974) asserts that "the organization of discourse can be studied only from within" (p. 26), he is not clear about the depth of immersion he expects. Even if we agree that a researcher should be ethnographically immersed in the classroom, we might wonder if it is good enough for a researcher to be familiar with a range of mathematics classrooms. Or, is it necessary for researchers to know well the particular classroom context of their participant students? I suggest that one of the strengths of my research is the extent of my knowledge of the researched classroom. I was present virtually all the time that the class met.

In another context, Stubbs (1974) considers the merits of "theoretical sampling," in which a particular form of language becomes more clear in an exaggerated context. Clinical interviews could be seen as *exaggerated contexts*. It is likely, for example, that Rowland's subjects communicated their reasoning more clearly in the interviews than they would in their usual mathematics classrooms. It is precisely because the students were removed from a familiar culture and interacting with a stranger that they could not presume shared meaning, and would therefore feel the need to say more than usual.

Cicourel (1970), a pioneer of ethnomethodology, outlines some rules for ethnographical research. His "*et cetera* rule" refers to a subroutine of interpretive procedure. The rule states that participants in a discourse have a unique ability

> to treat a given lexical item, category, or phrase as an index of larger networks of meaning [...]. The appearance of a particular lexical item presumes the speaker intended a larger set, and assumes the hearer "fills-in" the larger set when deciding its meaning. (p. 34)

In Rowland's clinical interviews, for example, participant children might not presume that their interviewer was fit to "fill-in" the meanings. Because they assume their listener's ignorance of the classroom culture from which they came, they could be expected to rely less than usual on the *et cetera* rule. They would explain themselves more carefully than they would in a familiar environment.

I am not suggesting that Cicourel's *et cetera* rule is antithetical to Stubbs' and Fairclough's promotion of ethnographic components in discourse analysis. Rather, I suggest that the discourse in an interview is significantly different from classroom discourse and that these differences are not all negative. And this points to a weakness of my research. Because of the close relationship I had with participants, they could be expected to rely on the *et cet*era rule. I did not benefit from the explanations that they would more naturally give an outsider. For example, the students explained themselves more clearly in conversation with David Pimm on the day that he visited the classroom.

Unlike Rowland, Morgan's (1998) intentions align with Fairclough's. She investigates student writing and teacher assessment of this writing to reveal which textual forms these assessors valued or favoured. Though there is much description in her work, her overall framework is critical. For various language features in student write-ups she considers a range of possibilities. For example, she considers that students' choices between an *I* voice, a general *you* voice and a passive voice will impact the evaluation of their work.

Morgan does not draw on a large corpus of data, but I suggest that this apparent shortcoming is not as serious as it would be in a descriptive study. She recognizes the limitations of her sample size and defends her choice by pointing to her overarching concern: "I do not claim to make any universal generalizations on the basis of the data collected [...]. Rather, my intention is to describe a range of practices" (p. 132). This account of her intentions follows Fairclough's tradition, his interest in finding a range of possibilities within a discourse.

Her corpus needed to be large enough to find a range of possibilities. However, it is possible to *imagine* the plausible without actually *experiencing* it. We can consider plausible alternative ways of participating in a discourse even if we have not experienced these ways of being. Teaching experiment studies are examples of this kind of boundary-pushing. Furthermore, it is worth mentioning a distinction between finding *a* range of possibilities and finding *the* range of possibility. "A range" suggests a sampling, and "the range" suggests an exhaustive list, which would be impossible to compile.

Like Rowland, Morgan could be seen to meet Fairclough's and Stubbs' ethnographic expectations with her general knowledge of the discourses in which her study was situated – both the classroom and assessment discourses. She demonstrates her awareness of the importance of context by devoting a chapter to it.

In the above consideration of Morgan's and Rowland's exemplary uses of discourse analysis, I suggested that the relative magnitude of the corpus of utterances under consideration may be smaller in critical analysis than it would need to be for descriptive studies. Whether the investigation is descriptive or critical, a degree of ethnography is necessary because of the context-specific nature of meaning. My extended interaction with the students and teacher in the researched classroom provided a very strong ethnographic component to the research, stronger than necessary for my critical orientation. And because of the *et cetera* rule, this strong ethnographic component left me to rely on my interpretations to "fill in" much that was not said.

Critical Language Awareness (CLA)

As described in the first part of this chapter, Morgan points to an alternative possibility for using critical discourse analysis. She suggests a need for what Fairclough (1992) calls *critical language awareness*. She expresses interest in the pedagogical value of such awareness. I have extended her call to engage mathematics students in critical discourse analysis to include a research opportunity.

In a paper Morgan presented to research peers, she seemed to lean toward promoting effective writing for students and toward critical analysis of classroom discourse for researchers and teachers. She began the paper saying:

> My primary concern in this paper […] is with the way in which language may serve as a crucial window for researchers onto the processes of teaching, learning and doing mathematics, where these are conceived of as social organised, that is, not only taking place within a social environment but structured by that environment. (Morgan, 2002, p. 1)

And in her closing remarks she pointed to a different goal for students:

> Greater awareness of the lexico-grammatical choices available within the semantic system and the meanings these may have in specific contexts may help mathematics teachers and students to develop more purposeful and hence more effective use of language. (p. 17)

I suggest the need for careful attention to the nature of pedagogical orientation. Is the focus within this orientation on possibilities for students or on possibilities for teachers? This question relates to one I am told Gattegno often asked: Is the teaching subordinated to the learning? Though teachers are the ones who typically access scholarship, the critical aspect of an investigation of possibility could concentrate either on possibilities for the teacher or on possibilities for students in the classroom discourse.

As Adler (2001) says, there is a dilemma in every classroom with regard to the level of attention directed toward language itself. The research I present here represents a first attempt to use deliberate prompts for language awareness in a mathematics classroom as a research opportunity. In addition to experimenting with possibilities for including CLA in a mathematics classroom, I drew upon student interpretations of language practice to complement my interpretations, which are coloured by a different context – a different set of priorities, experiences and conversations.

In preparation for this new kind of conversation, I have examined possible ways of initiating such a conversation between students, their teacher and a researcher (Wagner, 2003b). For this, I reanalyzed interview transcripts from my master's research. In these interviews, the participant students and teachers listened to recordings of teacher interventions during small group mathematical investigation. In addition to highlighting the roles of some features of language in mathematics learning, this investigation points to a particular way of structuring CLA in a mathematics classroom. It suggests the value of having students themselves do analysis of excerpts from their own discourse. The lessons I learned in this preparatory study were informative in the research presented in this dissertation.

My Relationship with the Discipline

In this chapter, I provided context for my interactions with the participant students and teacher in this research. As part of the scholarship that is included in this context, I briefly introduced the three kinds of agency that Pickering (1995) identifies as being at play in cultural extension. He characterizes the scientist's experience as a dance among *human*, *disciplinary* and *material* agency. This chapter's description of the scholarship that informed my interaction with the research participants is an account of my view of the *disciplinary agency* at play in the research. The following chapters will describe how I responded to the participants. My decisions were informed by the traditions of the various disciplines I considered relevant, but I needed to interpret these traditions within a particular, unique context. The intentions brought into the researched classroom, both by me and by the participants, formed the human agency. Using Pickering's dance image, which I will describe in greater detail in Chapter 5, the contribution of this research will come from the *dance of agency* between scholarly disciplines and the particular humans involved in the research.

Chapter 3 – Seeing What is Not There

In the first two chapters, I described the background that I brought to the research situation, including some relevant information about my life's path and some relevant scholarly literature. Here I focus on another aspect of the context of the conversation, the practical arrangements I made with research partners and the practical decisions I made regarding interpreting and synthesizing the data drawn from the relationship. With the classroom arrangements and my interpretive decisions, I was interested in opening up the possibility for me to see mathematics classrooms in a way that I have not seen them before. A strong feature of my approach is my decision to work closely with students, to listen to them. I believed that their unique perspective would help me see mathematics learning afresh.

Methodological Orientation

An account of practical arrangements is often referred to as method, and it is typically grounded in some tradition of inquiry. In this account, I too will refer to some methodological traditions, namely phenomenological, critical and narrative ones, but my overall approach was oriented to pedagogy, to the needs and expectations of the people with whom I was working. My background in the three above-named research traditions informed my pedagogical considerations.

With my pedagogical orientation at the forefront, I resisted the idea of ardently following a particular methodology. Valero and Vithal (1998) give accounts of two research studies in Columbia and South Africa, both of which experienced serious disruptions to the researchers' plans. With these stories, they note that dominant research paradigms are situated in Western cultures with assumptions of stability, predictability and consistency, situations that are less common in the rest of the world. Valero and Vithal show how the disruptions themselves are important data; researchers should not try to control, limit, or whitewash these disruptions. The assumptions they identify as basic to mainstream methodological paradigms are antithetical to possibility.

Valero and Vithal (1998) argue for closer attention to the context of the particular situation in which research occurs. Though they recognize the higher likelihood of disruption in relatively unstable environments, they point out the general instability throughout our rapidly changing world. For any context, especially one in which a researcher gives up many of the traditional controls, they highlight the importance of disruptions, the need to have a less linear and deterministic approach to method.

In my research, I allowed many people to influence what would happen. To highlight the contrast, consider for a moment the difference between a clinical interview and a regular classroom conversation. In the classroom, there are more participants, and each participant has her own agenda. With my choice to work in a classroom instead of in clinical interviews, I invited disruption by surrendering control. For instance, Chapters 5 and 7 are accounts of generative dialogue that emerged from students resisting my assertions in the classroom setting. In addition to promoting disruption by letting myself be outnumbered in the classroom, I was introducing a new element to the mathematics classroom, one that neither these students nor I had experienced before. Our on-going conversation was continually shifting as we came to grips with the nature of our discussion. In this unstable context, I took Valero and Vithal's (1998) advice seriously:

> When the research process is obstructed by uncontrollable disruptions emerging from the very same unstable nature of the social context and of the research objects that are considered, then

> the whole process of research has to be reconceived to allow the disruptions themselves to reveal key problems that should be addressed in order to understand, interpret or transform the real issues. (p. 157)

I needed to expect disruption, to welcome it. Indeed, as Valero and Vithal realized, the times when I felt most resisted were the most generative times, both for me and for the participant students. When the flow of events differed from my expectations, when my research was disrupted, I could expect the uncovering of possibilities I would not have seen otherwise. Valero and Vithal suggest *generativity* instead of *generalizability* as an alternative criterion for deciding which research is most worth attending to. In Chapters 5, 6 and 7, I give accounts of interactions that arose from such generative disruptions. In the situations described in Chapters 5 and 7, I felt the students' ardent resistance to my interpretation of particular language practices, and in Chapter 6, two students and I were distracted from my intended approach to language as we became interested in a paralinguistic phenomenon – the way people direct their gazes when communicating mathematics.

It is significant that Valero and Vithal (1998) provide a criterion for evaluating the worth of research. I used their criterion to decide which stories in my research were worth reporting. Though I resisted strong adherence to one particular tradition of research, I also recognize the importance of being aware of the traditions that I brought with me into the research situation, because my methodological orientation was significant in my decisions about what to attend to and what to write about. Heidegger (1967) describes the centrality of method:

> Method [...]decides in advance what truth we shall seek out in the things. [It] is not one piece of equipment of science among others

> but the primary component out of which is first determined what can become object and how it becomes an object. (p. 102)

Valero and Vithal, for example, suggest that disruptions are objects or events worth our attention. This particular orientation has its own peculiar challenges. It would seem that if I welcome disruption, then disruptions are not disruptive at all.

This peculiarity was important in the interpretation of my conversations with my student and teacher collaborators. I wanted students to argue with each other. I hoped for some tension between the teacher and the students. I wanted to be surprised. However, I did find myself frustrated about seemingly small things, like certain students' refusal to recognize a positive effect of the word *just* in sentences such as "Now we just solve it." Realistically, my frustrations pointed to the real disruptions to my agenda. They helped me see my agenda for what it was and attested to the importance of these disruptive situations to my research collaborators. Why would the participants put the effort into resisting if it were not important to them? I wanted to be open to such disruptions.

While openness was my goal, I admit that I was not always open to the events that took place. Burch (1991) highlights the importance of openness in research: "[M]ethod must lead us beyond what is already known of the subject matter in an everyday way to disclose it in its truth" (p. 20). With this criterion for evaluating the openness of particular methodological choices, Burch is not so much interested in the possibility of a research report's readers being led beyond what they already know. Rather, he is urging researchers to make methodological choices that open a space so that they themselves may be led beyond what they already know. In the next section of this chapter, I will give an account of the methodological traditions that informed my research. I want to show how these traditions made space for me to be led beyond the path I could have foreseen.

Burch (1991) suggests another criterion for selecting a method – the method should be appropriate to the topic of research. This criterion leads me to consider critical methodology, because I am investigating critical awareness in the classroom. Because the tradition of critical inquiry guides me to report possibilities as they arose out of the dynamic interactions among the participant students, the participant teacher and myself, I choose to tell the story of the conversation. The format of this telling directs me to be mindful of the emerging tradition of narrative inquiry.

In addition to the critical and narrative inquiry traditions, I also use a third orienting tradition – phenomenology. Heidegger's (1967) and Burch's (1991) guidance are part of the phenomenological orientation that has informed my view of method. They and others in that tradition have helped me be more critical of the way I comprehend (or apprehend) the phenomena that seemingly appear before me. The phenomenological tradition has prompted me to be mindful of connections among the presuppositions I brought to the inquiry, my experience of the situation as it presented itself to me, and my languaged account of my understanding and interpretation.

Critical Inquiry

Skovsmose and Borba's (2000) methodological framework for critical mathematics education research shares Burch's (1991) orientation to being open to new things. They characterize the principal interest of critical research as possibility:

> [I]t seems to us that it is by no means a simple truth that research should deal with what *is*. As a first simplified characteristic we will suggest that doing *critical research* means (among other things) to research *what is not there* and *what is not actual*. (p. 5, emphasis theirs)

Despite the inherently inductive nature of such research, it is important to make plans. But such plans are different from plans for typical research within a positivist paradigm, research in which factors are controlled and complexity is constrained.

There are obvious connections between the framework provided by Skovsmose and Borba (2000) and the paradigm for critical discourse analysis detailed by Chouliaraki and Fairclough (1999) and described in Chapter 2. Like their discourse analysis counterparts, Skovsmose and Borba are not interested in *describing* what already exists. They support the investigation of new possibilities.

Skovsmose and Borba (2000) use the diagram shown in Figure 3-1 to illustrate the activities of a researcher engaged in critical inquiry. There are three situations among which the researcher makes translations. The current situation (CS) is manipulated to set up an arranged situation (AS) as the researcher engages with classroom participants in exploring an innovation. As participants and the researcher interact, each person imagines a situation that he or she wants or expects the arranged situation to become. The current situation and the imagined situation (IS) may be seen as positive or negative. Indeed, it is likely that participants would see aspects of any current situation as being negative. These would be things the participants would want to change. The arrangements made to rectify the current situation are called the arranged situation. Thus, arranged situations must be seen as positive, because they carry the participants' intention for improvement. The imagined situation can include some negativity, because participants can expect things to go wrong when establishing the arranged situation.

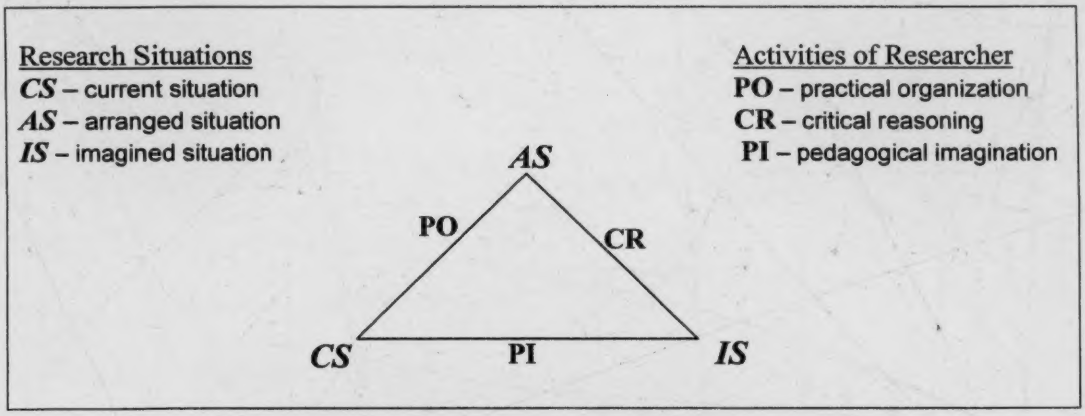

Research Situations
CS – current situation
AS – arranged situation
IS – imagined situation
Activities of Researcher
PO – practical organization
CR – critical reasoning
PI – pedagogical imagination
AS
PO
CR
CS
PI
IS

Figure 3-1. Illustrating critical research methodology (from Skovsmose and Borba, 2000, p. 12)

Skovsmose and Borba note that participants are involved in research as well. I add that the participants' research involvement can be expected to be characterized by different agendas and levels of commitment; I worked together with students and their teachers to explore possibilities for language awareness in the mathematics classroom, but we all had other concerns as well. The work of researchers and participants involves translating between the three categories of situations. *Practical organization* (PO) marks the transformation of a current situation into a new arranged situation. As we critically reflect on the new situation, we think about what might become possible or probable. In this, we are doing *critical reasoning* (CR). None of these situations is static. One day's arranged situation is the next day's current situation. For example, the participant teacher and I were constantly thinking about the current situation in terms of our respective imagined situations. Skovsmose and Borba (2000) call such considerations *pedagogical imagination* (PI).

Though participant students were similarly engaged in imaginative work, their imagination was more oriented to learning than to pedagogy. However, I do not want to underestimate the participants' interest in and input into their teachers' pedagogical decisions. Typically, students influence the pedagogy in a classroom more subtly than their teachers do, but this does not mean that there is no influence. Because the quality of their learning is at stake, they should be expected to have a significant interest in their teachers' pedagogical decisions. Before I began the conversation with students in this classroom, I predicted that they might even be sympathetically interested in the impact of our inquiry on other students through the medium of my publications and presentations. Evidence of this kind of concern did appear in the research. For example, as reported in Chapter 7, a student strongly articulated a concern for the self-esteem of students in general though it was clear that he himself did not suffer self-doubt in response to the language practice that prompted his concern.

In this research, the "current situation" was unknown at first. Once I began conversations with the teacher and students, we encountered together an ever-evolving "current situation." Each day, the class experienced a new current situation, and we worked together to explore our imagination and move toward new arranged situations.

Before entering the conversation with the teacher and students, all I could do was describe *my* imagined situation. Because imagination is significant work in critical inquiry, it is important for me to be aware in my interpretation of the imagined situation I brought with me into the classroom. My imagined situation directed my interaction with participants as we negotiated arranged situations. The experiences the participants brought to our interaction, together with our experiences of the interaction itself, continued to influence our imagination and our planning. Nevertheless, the imagination that I brought into the research relationship was especially significant because it set the tone of our interactions within it.

This recurring cycle of current situations being adjusted according to our imagined situations and forming new current situations was interesting in itself, but the story of its development is not the goal of my research. My goal was to explore possibilities for classroom discourse. In the coming chapters, I will describe two kinds of possibilities. Events in the researched classroom may be seen as possibilities for other classrooms: because these things happened once, they could happen again in different contexts. At least, something similar could be expected to happen. These events could not possibly recur *exactly* has they happened in the unique context of my researched classroom. Another order of possibilities that I will be reporting was prompted by the events in this classroom. There are possibilities that I can now imagine even though they did not actually occur in this researched classroom.

Narrative

An experienced educator and friend in Swaziland once said to me that people are more likely to reconsider traditions or their own behaviour when confronted with a story. Story-telling is much more potent to effect change than are logical argumentation or statistical "proofs," he asserted. If we try to convince people to interpret their experiences in a new way, we can expect resistance. By contrast, if we tell a rich story, we provide, in a less threatening way, a new experience that they may be prompted to reconcile with their other experiences. If they are ready to change, they will change, said my Swazi friend. If they are not ready, they can at least enjoy the story as they would enjoy a play or a novel. When these not-yet-ready people become ready to learn, they can draw upon the story as they revisit it in their recollection. If the audience is ready only to fight, they will fight no matter what we do, concluded this friend. Do not worry about them, he advised. There is nothing we can do about them. This friend's advice was concerned with making a positive impact on a group of people. I hope that my various accounts of this research afford my audiences the range of possible responses described by my Swazi friend.

The role of this account and other accounts that I give of this research is to seed thoughtfulness and awareness of new possibilities. Marmon Silko (1996), a lawyer and advocate for native land claims in the United States, writes: "I decided the only way to seek justice was through the power of the stories" (p. 20). She does not *impose* justice; she *seeds* it through stories. After a seed is planted, the gardener needs to be patient and let the seed's environment take over. Similarly, I will not be saying how mathematics classrooms ought to be. I will only be giving accounts of possibilities. I hope for these seeds of possibility to grow in their own way when planted in contexts that I cannot imagine or understand.

Part of the unique nature of the story I tell is my foregrounding of students' interpretations. In addition to seeing myself as an interpreter of classroom language practice, I tried to tease out students' interpretations. I found that it is difficult to draw out the voices of people who have been silenced for so long – the students, in this case. Indeed, as described in Chapter 7, when students spoke with a strong voice, I sometimes tried to argue against their interpretations even though my resistance to them contradicted my intentions.

I believe that students have truth to tell us, truth that has been repressed in our research and education traditions. I do not intend to turn the tables and suggest that students should have a more powerful voice than researchers and teachers. Rather, I consider it important that students' experiences be seen as significant in mathematics classroom discourse, important for the students and important for people who want to understand the classroom.

My assertion that there are repressed truths waiting to be heard suggests a belief in a multiplicity of truths, a view that challenges research traditions that look for answers more than for questions, for accurate description rather than for imagined possibility. For me, repressive political regimes exemplify problems with one-truth beliefs. They exemplify the problems with answers. In the context of post-Apartheid South Africa, for example, poet Antjie Krog (1998) resists the kind of truth she has been fed from infancy:

> The word "truth" makes me uncomfortable. [...] I hesitate at the word; I am not used to using it. Even when I type it, it ends up as either *turth* or *trth*. I have never bedded that word in a poem. I prefer the word "lie." The moment the lie raises its head, I smell blood. Because it is there ... where the truth is closest. (p. 50)

I am reminded of Crites' (1979) warning that narration is ripe for deception, and that deception is intimately connected to self-deception: "[T]he aesthetic formation of experience offers resources for truth-telling as well as for self-deception" (p. 128). I employ narrative tools for their potential contribution to truth telling, and assert that "truth telling" in other forms of inquiry and writing is equally susceptible to self-deception. Crites agrees: "[A]nyone who supposes that he has won his way to permanent immunity from self-deception, by *whatever method*, has not learned much from experience at all" (p. 128, emphasis mine).

Even the way I weave together the stories from the research conversation is significant to the truth that I tell. Carter (1993), in her consideration of story in the study of teaching, reminds that stories are strung together by imposed causality, they "are constructions that give a meaning to events and convey a particular sense of experience" (p. 8). In my interpretations of the conversation, I imposed causality that was not necessarily there. I did this in order to make sense of the situation. But, I am aware that I only saw a small part of the story, even though I had significant access to the situation.

Whenever I recall an event or a series of events, the retrospective sense I make of this situation is necessarily different from the moment under consideration. In the moment, I *experience* the situation. In reflection, I look for connections and explanations. I think of the accounts in this dissertation as an "explanatory fantasy." This term is inspired by Morgan's (1998) description of teacher–assessors' readings of students' mathematical writing. She calls their explanations of the students' mathematics "explanatory narratives." These assessors could only imagine what connections the student–authors made between each line of written mathematical symbols. Similarly, in this research, though I was present for the events I am describing, I could not attend to all aspects of the classroom. In addition to this limitation, my memory and records represent mere selections of the countless things I noticed while experiencing the classroom. Looking back, I can only reconstruct on the basis of artefacts – my memories, my notes and my audio and video recordings.

Bakhtin (1970/1986) illustrates the transformational effect of retrospective accounts using an old joke he heard as a child:

> There used to be a school joke: the ancient Greeks did not know the main thing about themselves, that they were *ancient* Greeks, and they never called themselves that. But in fact that temporal distance that transformed the Greeks into *ancient* Greeks had an immense transformational significance. (p. 6)

The narrative inquiry tradition shares with critical inquiry esteem for participants as co-researchers. Skovsmose and Borba (2000) point to the non-sense of seeing participants as research objects. They are subjects, not objects. If participants are objectified, their critical imagination is lost. Clandinin and Connelly (1988) note that in narrative inquiry both the principal researcher and participants, whom they call *collaborative researchers*, ought to be "meaningfully involved in the same problem" (p. 272).

Though I hoped that my participants would become engaged by the problems in which I was interested, I knew their perspective on these problems would be different. They would have a different set of problems from mine. Student participants wanted to pass the course, or to achieve a certain grade level. The teacher participant wanted students to understand the mathematics in the curriculum and needed to assure parents and administration that curricular expectations were being met. I wanted to talk with them about their discourse, and I wanted especially to listen to them engage in analysis of it.

As we worked together and related with each other, we shared each other's concerns. I too wanted students to pass and all stakeholders to be satisfied with the curricular coverage in this class, but my principal concern, the concern that brought me to the class, was related to my research question. Similarly, student participants would have a stake in my research questions, but their principal concern was their mathematics learning or their performance in the class.

It may seem that each participant's unique concerns would conflict and interfere with those of the others, but it is exactly this diversity of perspectives that brings depth to the compiled account I bring forward. I could not ignore the students' need to address particular outcomes mandated in their program of studies, because these were their principal concern. Just as any story can be interpreted in multiple ways, any event can be interpreted in diverse ways. The unique concerns and prior experiences of each participant researcher contributed to the quality of my summative account.

The first two chapters tell of the stories I brought into the research conversation, but the participant teacher and students' stories are less known to me. Though I have already talked about this context, I also address it here, to acknowledge that my awareness of this background is part of the method with which I engaged in the research conversations. This context was behind my choices for action and my interpretations. Though I do not like the idea of pasting in theory as a pretext for my inquiry after being engaged in the inquiry itself, I recognize that some theory is part of my pretext. Scholarship in the areas of mathematics education, discourse analysis and language awareness formed part of the experience that I brought into the inquiry. It was part of the context of the research site because it was a part of me.

Research Participants

The teacher and students with whom I conversed also merit introduction. They too were part of the context of the conversation. Their backgrounds and their expectations are significant to the account I give. However, their background is less available to me. Before introducing these significant people, I will describe how we came together.

When considering what kind of class would be ideal for uncovering a range of possibility, I wanted an ethnically diverse student population and a confident participant teacher who would not be shy about resisting my pedagogical suggestions when appropriate.

The teacher, whom I will call Mrs. Cheryl Hill, is a former science teacher who has taught high school mathematics for more than a decade. This name and all others referring to participants are pseudonyms. In this dissertation, I will refer to the teacher as Mrs. Hill, instead of Cheryl, to signify that her role was different from that of the students. The more formal address points to her authority in the classroom.

The principal of the school recommended Mrs. Hill because of her impeccable reputation. When her principal suggested her, I was pleased because I had heard her contributions at professional functions, and had had some minor interactions with her. I knew she had both confidence and a strong knowledge base. I would not be able to bully her into doing anything. I saw this as a positive thing, because I wanted to work with someone who would be strongly oriented to pedagogy. I expected that I too would be oriented to pedagogy, but I worried that my research interest might at times eclipse my concern for the students. Ainley (1999) and Phillips (2002) both write about this concern when a researcher plays multiple roles. Mrs. Hill also seemed pleased to be working with me when the principal introduced us.

I thought students with a range of ethnic backgrounds would prompt a wider range of perspectives in our conversations, but I was worried that girls from some ethnic backgrounds would tend to be quiet. This prejudice was based on my experiences in African and Asian countries, where I have seen women and girls marginalized. Most of my teaching in Canadian high schools had been in rural Alberta, where there was little ethnic diversity, so I was not sure what to expect from young Canadians with Asian and African backgrounds.

The research class started with thirty-four students, but two withdrew quite early. An exchange student from Germany withdrew because of scheduling requirements and a girl was required to withdraw after she beat up a friend of some girls in this class. Of the remaining thirty-two students, there were fifteen boys and seventeen girls. The girls often significantly outnumbered the boys because the boys' attendance was generally less dependable.

A significant number of these students were new to Canada. Figure 3-2 lists some of the students. The information in it comes from the students themselves, from what they told me. I did not check it against any other records. Other students may have been recent arrivals too. Most of these new arrivals spoke English that was indistinguishable from that of their peers. Tim and Shaun were exceptions; they had a difficult time with English. Also, the country of birth for some these individuals was often not as significant to them as other countries in which they had lived, including Canada and other intermediary places.

Student	Sex	Country of Birth	Years in Canada
Arwa	F	Tanzania	?
Lisa	F	China	?
Priscilla	F	Rwanda	< 1
Sita	F	Zambia	?
Tharshini	F	Kenya	?
Joey	M	Malawi	3
Shaun	M	China	< 1
Signot	M	Russia	3
Tim	M	Korea	2

Figure 3-2. Some students' backgrounds

Other students in the class would be labelled by some people as ethnic minorities, but I resist such labels because I sense that such labelling is too reliant on skin colour and facial features. As far as I know, all of the students in this class came from immigrant families, families who have been in Canada from one to multiple generations, having come from Europe, Asia and Africa. None of them had indigenous ancestors. I divulge these students' background information to show that they displayed the kind of diversity that I hoped for. I feel most comfortable in such settings of rich diversity.

In addition to the diversity of perspectives that was likely to be engendered by this ethnic diversity, there were language complexities. As stated in Chapter 2, Adler (2001) and Barwell (2002) have noted particular classroom and research issues that are exacerbated in multi-lingual environments. I once heard a research report in which the presenter said his researched classroom was composed mostly of ethnic minority students. I suggested to him that these students were therefore not the minority. They were the majority. However, there is a sense in which he was right to say they represented a minority. For example, if these children were predominantly Mandarin-speaking, they were probably still taught in English, the first language of the literal minority. Because Mandarin would be ignored in such a setting and English would dominate, the Mandarin culture and language would be relegated to a minor, if not negligible, role. English would occupy the majority role.

In Barwell's (2003) challenge to the mathematics education scholarship's English-dominated community, he differentiates between three kinds of multi-lingual classrooms. The mostly Mandarin classroom described above and my researched classroom would each be classified by Barwell as a *monopolist discourse*. One language is used in the classroom, though other languages are available to the students. In my researched classroom, for example, many of the students were fluent in more than one language, whether they were recent immigrants or not. I asked Joey and Gary on one occasion, and Arwa and Tharshini on another occasion, what they thought about the significance of their other languages. Joey and Gary responded with vague references to mathematics being a universal language. Arwa and Tharshini said it was an interesting question that they had no answer for at the time.

In addition to my preference for an ethnically diverse student body, the school needed to be large so that the research class could be concurrent with another section of the same course. I thought this would afford students a real choice between participation and exclusion. In our planning, the principal of the school promised to overload the research class, so that students who opted out of the research could migrate to the other section and not cause imbalance. However, this balancing manoeuvre was not accomplished because of other administrative concerns irrelevant to the research, and perhaps because both sections were extraordinarily large in relation to typical class sizes in previous years.

The teacher, Mrs. Hill, described for me some other significant features of the class demographic. Many of the school's highest achieving students were enrolled in the school's International Baccalaureate (I.B.) program. These students took special courses that fit the program's needs. Pure Mathematics 20 would normally draw the students who intend to matriculate into university, generally the higher achieving students. However, because of the special program, most of the highest achieving students took the special I.B. pure mathematics classes. That left a median group for Pure Mathematics 20. Generally, these students were not the highest achieving in the school, but they were academically inclined.

Another feature of this particular section of Pure Mathematics 20 was its high proportion of "problem students." A month into the semester, Mrs. Hill divulged to me that this section included three of the school's worst discipline problems. This situation seemed to be a chance occurrence, but it was significant to the interaction in the classroom. Compared with their peers, these three "problem" students participated much more in our discussions about language in the classroom. One of them, Joey, stopped coming to class once he knew he was going to fail the course, but he continued to make efforts to meet with me to talk about language practice.

Ethics Procedures

The students' parents were also relevant to the research. Because the students were too young to be legally responsible, their parents needed to provide written consent for their children to be involved in the study. On the first day of class, Mrs. Hill and I informed the students that we would both be teaching in their class. After she distributed the course outline and discussed classroom rules of behaviour, I introduced the research situation that I wanted to establish. When I asked if they had any questions, Darren asked if the discussion about language had anything to do with mathematics, and Joey asked what I would be doing with the research. I distributed letters to the students' parents with permission forms for them to sign (see Appendix A). Also attached was a flyer inviting the parents to an information evening two days later.

Nine parents came to the meeting, and two were parents of the same child. One student, Mansoor, came to the meeting with his mother. Other than Shaun, who did not seem able to communicate in English, Mansoor turned out to be the quietest, most reserved student in the class. Mrs. Hill and I planned the meeting to be no longer than an hour. She introduced herself and me and then I outlined the research plan, basically following the permission letter. This took less than fifteen minutes. Then we asked for questions. The parents' questions were not related to the research. They asked about the Pure Mathematics curriculum and how it would prepare their children for university. This had been a hot topic in the popular press with recent curriculum changes. This issue is outlined well by McCabe (2000). In an informal chat over cookies, Trina's father asked me about research methodologies. He was aware of tensions between quantitative and qualitative methodologies and thought it interesting that a mathematician would choose qualitative inquiry.

Jessye's mother wanted to come to the meeting but had a time conflict. Instead, she phoned me to ask her questions. We connected by telephone the following Monday, after four days of classes. Jessye had wanted to switch to the other section because she did not want any "weird practices" to interfere with her learning of mathematics. I described for her mother the kinds of tasks I would give these students and her concerns were alleviated. By then, Jessye had already decided that she was content to stay. They both recognized the unique advantage Jessye was afforded by having two experienced and respected mathematics teachers.

Contrary to what Mrs. Hill and I had promised the students, a vice-principal had resisted Jessye's plea to switch sections and had asked Mrs. Hill to try to convince Jessye to stay. At the time when he made the request, he did not say who the student was. Though Mrs. Hill and I were uncomfortable with this turn of events, we had no power to overrule the vice-principal. We guessed that it was Lisa who had concerns because she had said that she did not want to be on videotape. Mrs. Hill asked Lisa to stay after class on the first Friday and asked her if she had any concerns about staying in the class. Lisa was happy to stay but still wanted to stay off videotape. When I later talked with Jessye's mother, I found that it was not Lisa, but Jessye who had asked to be transferred out of the class.

Lisa was not the only one who wanted to stay off videotape. On the permission form, I asked parents and students to check off which of the following they approved:

☐ to be videotaped in the mathematics class;
☐ to be audiotaped in the mathematics class;
☐ to be interviewed;
☐ to allow his/her work to be photocopied for the purpose of the research.

(excerpted from the consent forms; see Appendix A)

Two other girls did not check off the box to indicate their willingness to be videotaped. To accommodate these individuals, I set up the videotape on a tripod in the same spot each day. I told the class where the camera's blind spot was and invited them to sit in it any day they did not want to be on tape. A number of girls were shy about being videotaped and sat there sometimes. There were eight to twelve desks in this blind spot, and usually they were all occupied. Four students said at the outset that they did not want to be interviewed, and others opted to decline invitations to be interviewed during the research, a phenomenon that I describe in the next chapter. Two students did not want to be audiotaped. Only Lisa did not want me to make copies of her work. I respected these people's wishes. For example, when I noticed a girl who did not consent to being videotaped get up to walk to the front of class to throw something in the garbage, I stood in front of the video recorder. Because these students were told that I would ask for special permission to show any particular video segment, they must have had other reasons to want to avoid being recorded.

As I waited for students to bring in their signed permission forms, I took field notes each class. It is not uncommon for high school students to take a long time to bring in signed forms, but collecting this class set of permissions took longer than I expected. One particular form took a very long time to come in. The boy was a "home-stay" student whose parents remained in another country. He lived with a Canadian family who were not his legal guardians. His legal guardian was an administrative officer, someone this boy rarely saw.

On the Monday of the fourth week of classes, I began videotaping. The last permission form had been brought in the previous Thursday, but I wanted to tell the students on Friday that I would begin taping on Monday so they would not be surprised. In addition to delaying the beginning of my videotaping, I did not introduce explicit discussion about language practice until I had ethical clearance from everyone. I continued to write field notes every day after I began videotaping. I did not transcribe all the video material, only episodes I thought I would use.

I stopped videotaping classes three weeks before the class ended because I had promised the participants I would show them transcripts that I thought I might use in reports of this research, including this dissertation, journal articles and conference presentations. In these last three weeks, during which I was not regularly present, I visited the class periodically to let the participants see transcripts and proposed conference papers. I even showed some students drafts of two full papers that I would be presenting at conferences (Wagner, 2004a; 2004b). During this time, I also conducted a few interviews with some students.

Further to the classroom dialogue, I also interviewed students and Mrs. Hill in various ways. I audiotaped these interviews. I arranged interviews with students for lunch times, but about half of the time the students who had agreed to come in did not show up for the interviews. In response to this problem, I interviewed students in the hall during class in times when they were not engaged, usually after they finished writing a test. My chats with Mrs. Hill before and after class were more generative than my audiotaped interviews with her. She seemed to be more careful about what she said in interviews. I will report on these interviews more in the coming chapters.

Developing Relationships

The class met every weekday for sixty-six minutes, except for Tuesdays, when we met for a double block, two sixty-six minute classes with a two-minute break in the middle. Before the term began, Mrs. Hill suggested that we alternate teaching days. She usually taught on Mondays, the first block on Tuesdays, and on Thursdays. I taught the remaining blocks: the second block on Tuesdays, and on Wednesdays and Fridays. We followed her year plan, which included summative tests at the end of each unit.

In my preparations, I did not foresee the frustration I would experience with teaching. I found myself trying to fit Mrs. Hill's style in the classes I taught. A typical day began with the teacher showing students how to do homework questions that they had trouble with. Next, the teacher would give notes, which would include definitions and example questions. After giving an example or two of a particular kind of question, the teacher would let the students try one on their own, and then this question would be discussed.

Typically, there were one to three sets of examples like this. After these examples, the teacher would assign some seatwork/homework questions for students to do in the remaining portion of the class. During this exercise time, the teacher would circulate amongst the students, giving them clues and hints, answering questions they had, and trying to encourage or scold the students to "get to work." In the last few minutes, students would close their books one by one and migrate toward the classroom door, waiting for the bell.

This typical structure was modelled first by Mrs. Hill, and it seemed to me she expected the same from me. Because we followed her outline, each day required a particular mathematics curriculum outcome to be addressed. And she expected me to provide examples to students of every "type" of question. She had a binder with her Pure Mathematics 20 notes in them. For every outcome, she had written the definitions and example questions she would transfer onto the overhead projector when she taught.

This style was familiar to me, because she structured classes in much the same way as I had when I taught high school full time. However, as a result of my reading and research in the last three years, my perspective on mathematics teaching has changed. There was no doubt in my mind that I would have taught much differently if I were a regular mathematics teacher. Mrs. Hill repeatedly told me that I should teach however I wanted to, but with her outline that required an outcome addressed each day and her expectation that students be given examples, there was little time to do dwell on anything else – for example, to investigate a rich mathematical task, as I had students do in my master's research (Wagner 2002a).

Despite my sense that I was leading classes in the same traditional way that Mrs. Hill was, she noticed a significant difference in our styles. At the end of the semester, she noted her own drift toward what she thought of as my style. She found herself drawing students into conversations about the mathematics much more than she would normally.

In addition to the adjustment both of us faced working with a strong peer in the mathematics teaching, something neither of us had done before, we had to develop a way of incorporating conversations about language, which was the whole point of our coming together. There were times during the term when I made intentional shifts in the way I tried to prompt these conversations. I will describe these in the next chapter, as I give an account of the students', Mrs. Hill's and my developing critical language awareness.

Practicalities of Telling the Story

In addition to making decisions about the overall arranged situation for this research, I made decisions during my interactions with participants and after these interactions. Chapter 4 will give an account of decisions I made during the research in response to the situation as it presented itself. Bur first, in the next section of this chapter, I outline some of the decisions I made about presenting my interpretations of the researched situation.

The Data

The foremost decision I faced was to select and connect together events to tell worthwhile stories. In accordance with what I told the research participants, I chose as primary data our interactions that related explicitly to language. In our dialogue about language, we drew upon a secondary set of data – video and audio taped recordings and transcripts of class discourse that was explicitly focused on mathematics and learning mathematics.

Most of the text I chose as objects of analysis in conversation with students was composed predominantly of oral utterances. I chose these oral interactions because of the overall form of the discourse in this class. Other than the heavily symbolic notation that characterized almost all of the students' writing, most of their utterances were oral.

Most of the transcripts in this dissertation represent conversation *about* mathematics discourse. The relatively few transcripts of mathematics discourse itself are provided as context for my accounts of discussions about language practice, because the participants and I referred to these artefacts. For example, Figure 4-2, in the next chapter, details a transcript of an episode of mathematics teaching and learning. It is a transcript that I showed the students as part of the stream of conversation described in Chapter 4.

While my on-going conversation with the participant students and Mrs. Hill forms the primary data in this research, this body of conversation needed to be broken up into manageable bits for the telling of the story. I chose three major threads of the conversation as examples of the kind of interaction that can arise from attention to language in a mathematics classroom. These strands, which are described in Chapters 5, 6 and 7, illustrate some possibilities uncovered in this study of critical language awareness. However, these threads of conversation were not representative of the nature of the participants' response to language awareness prompts. For this reason, I include a chapter that describes their overall response (Chapter 4).

The threads of conversation described in these four chapters are also divided into smaller bits. Indeed, the conversation could be divided and divided, continually broken down into smaller and smaller bits. I chose to consider the utterance as the fundamental unit of language for analysis, following Bakhtin's (1953/1986) advice:

> [T]he *real unit* of speech communication [is] the utterance. For speech can exist in reality only in the form of concrete utterances of individual speaking people, speech subjects. [...] The boundaries of each concrete utterance as a unit of speech communication are determined by a *change of speaking subjects*, that is, a change of speakers. Any utterance – from a short (single-word) rejoinder in everyday dialogue to the large novel or scientific treatise – has, so to speak, an absolute beginning and an absolute end: its beginning is preceded by the utterances of others, and its end is followed by the responsive utterances of others. (p. 71, emphasis his)

Like Bakhtin, I use the word *utterance* to refer to a spoken or written text that begins when a person begins to speak or write and continues until that written or spoken text is terminated or interrupted. Using this understanding of the utterance, this whole dissertation comprises a single utterance.

Presenting Data

The transcripts that I present in this thesis' account of the research demonstrate the value of choosing the utterance as the fundamental unit. Consider the alternative of using the *sentence* as the basic unit of language. Do people speak in sentences? When transcribing from the audio records of the classroom discourse, I became increasingly aware of the lack of formal sentence structure in the students', Mrs. Hill's and my oral utterances. Indeed, most students' initial response to seeing printed transcripts of our interviews was embarrassment regarding their poor grammar. My attempts to alleviate their concern by directing their attention to my own poor grammar seemed unsuccessful.

Just as I impose causality in the telling of the story from the researched classroom, I imposed causality when formulating sentence structure in transcribing oral utterances. I did this by inserting punctuation – periods, commas, em-dashes, exclamation marks, and question marks. Again, to illustrate my rationale I consider alternatives. Instead of using periods and commas, I could have used special coding to show the length of each pause. Instead of using exclamation marks and question marks, and thus imposing intention on relevant utterances, I could have used a more objective encoding that marks the dynamism of intonation, volume and other prosodic features of the speech. In addition to the problem of choosing which prosodic features merited reference in any given utterance, there is the problem of distracting the reader of the transcript. With the inherent difficulty of reading specialist coding fluently, the reader could not be expected to attend to the utterances themselves. By choosing the most familiar representation possible – conventional punctuation and square-bracketed blocking similar to that in play scripts – I believe that I have represented the utterances the most clearly.

Another problem of transcription can be addressed by focusing on utterances as complete units. When listening to audio records, many words are difficult to comprehend. The background noise, a speakers' poor diction, or poor positioning of the recording device could obscure individual words, series of words or even complete utterances. Again, to aid the reader, I have not included any coding that represents obscured speech.

It is not uncommon for transcripts to have symbols that represent indecipherable speech, but whenever I read such transcripts I wonder where the transcribers drew the line between the indecipherable and the unclear. At times, in my transcription, I knew a word could be only one or another. Sometimes I could discern the actual word by context. How different is this from what people do in ordinary conversation when we discern what a word must have been by context even when the word is utterly indecipherable because of the stamping of a foot, for example? My point is that we make judgment calls all the time when we listen to speech. Indeed, no two individuals pronounce a word exactly the same way. Just as meaning is as dependent on context as it is on word choice, our hearing of words is as dependent on context as it is on the actual sounds made by a speaker.

For example, the word *because* was common in the transcripts included in and excluded from this dissertation. This word was said with varying stress on the initial syllable *be*. Sometimes it sounded as if a person said *cuz* instead of *because.* Sometimes, the person said *be-cuz* with a very faint but discernable *be*. Sometimes both syllables were prominent. In each of these cases, the context made it clear that the person was saying the word *because*. I have chosen to represent all such cases with the complete word *because*. Instead of presenting my participants' speech in its *bastardized* form, I have chosen to present it in a *legitimized* form. However, in the transcripts presented as figures (e.g. Figure 4-2), which are copies of transcripts I showed students, I did not follow this protocol. These transcripts are presented as the students saw them, before I had finalized my decisions about transcript presentation for this thesis.

Another dilemma I faced regarding transcription could not be addressed by Bakhtin's unit of analysis – the utterance. Frequently, people begin speaking when others are not finished speaking. Furthermore, people very often speak simultaneously. Bakhtin's definition of an utterance assumes clear turn-taking. It was not uncommon in this research for students to complete each other's sentences. Arwa and Tharshini, in the stream of conversation reported in Chapter 7, did this especially often. In my transcriptions, I represent situations of simultaneous speech with vertical lines bounding the simultaneous portions of speech. Where space permits, I include a square-bracketed note – [*simultaneous*].

There are other problems with Bakhtin's definition of the utterance. In Chapter 4, for example, I will differ from Bakhtin when I suggest that silence can be an utterance. I will also suggest in Chapter 8 that an artistic or geometric image can be an utterance. If an image or an instance of silence is taken as a response to an utterance and the image or the silence is responded to by other utterances, it should be taken as an utterance as well.

Seeing Like Children

In this chapter, I gave an account of some key structuring decisions that influenced the formation of this dissertation. To sum up my sense of the choices I made in the structuring of this research, I refer to Bakhtin (1970/1986), who emphasizes the importance of intersecting cultures. The significance of the on-going conversation in the researched classroom rests upon the insertion of a scholarly tradition and its orientations into a pedagogical tradition and its orientations. Though the mathematics classroom is not a foreign environment for me, it was made strange by my sense of belonging to the scholarly community:

> There exists a very strong, but one-sided and thus untrustworthy, idea that in order to understand a foreign culture, one must enter into it, forgetting one's own, and view the world through the eyes of this foreign culture. This idea, as I said, is one-sided. Of course, a certain entry as a living being into a foreign culture, the possibility of seeing the world through its eyes, is a necessary part of the process of understanding it; but if this were the only aspect of this understanding, it would merely be duplication and would not entail anything new or enriching. *Creative understanding* does not renounce itself, its own place in time, its own culture; and it forgets nothing. In order to understand, it is immensely important for the person who understands to be *located outside* the object of his or her creative understanding – in time, in space, in culture. For one cannot even really see one's own exterior and comprehend it as a whole, and no mirrors or photographs can help; our real exterior can be seen and understood only by other people, because they are located outside us in space and because they are *others*. (pp. 6-7, emphasis his)

Students interact with mathematics teachers every day throughout the world. The thing that set this research apart from everyday classroom occurrences was my dual role as researcher and teacher. Because of this role, I was able to prompt conversation that would not occur otherwise. And because I was immersed in the classroom, I was afforded an opportunity unique in educational research. I was able to listen to the students in their environment. I heard what they had to say about issues that are typically addressed by scholars alone.

> [A]s long as we see the actions of little children through the models of our shared adult conventionality, we are not likely to see the world as children, in their own uniqueness, see it.
> (Beekman, 1983, p. 40)

My methodological choices were motivated by my orientation to the voices of the children, who ought to be seen as the principals in any mathematics classroom.

Chapter 4 – Hearing Silence

And in the naked light I saw
Ten thousand people, maybe more.
People talking without speaking,
People hearing without listening,
People writing songs that voices never share
And no one dared
Disturb the sound of silence.
(Simon, 1965/2001, track 1)

The sounds of silence became significant in my conversation with the participant students in this research. In this chapter, I give an overview of my attempts to raise their critical awareness of language. After outlining the evolving nature of my critical language awareness (CLA) prompts, I consider these conversations in terms of *kept silences* and *broken silences*.

Passive Resistance in the Researched Classroom

In my consideration of method (Chapter 3), I expressed my interest in the resistances to my research efforts. I agreed with Valero and Vithal (1998), who assert that disruptions are significant in research, as they often point to the heart of the situation being researched. In the three chapters that follow this one, I will recount three generative times of relatively active resistance. While it is tempting to find active resistance the most interesting, I characterize most of the student responses to my prompts for considering language as *passive resistance*. This is not to say there was no response to my prompts. Even silence can be a response. After outlining the development of our language awareness conversations, I will consider the research relationship in terms of different kinds of silences.

Throughout the term in which the researched class met, I shaped each new language awareness prompt in response to students' responses to previous prompts. Most of the time I felt as though the students gave polite, shallow, uninterested answers to the *general questions* I raised about language. By contrast, their responses were more disparate when they responded to my presentation of specific *classroom texts* and my assertions about these texts. In these situations, students were more likely to be literally silent, or, in exceptional cases, to engage spiritedly in conversation about language and mathematics.

The nature of our dialogue about language was new both to the students and to me. Together we had to develop a new genre of class discussion, a new way of talking with each other. We were developing not only a new means of interaction, but also content that was new to all of us. Furthermore, there was a close connection between means and content because the new content was explicit conversation about the means of our communication. When I consider this new conversation as a genre, I follow Bakhtin (1953/1986), who recognizes genres in a wide variety of forms, from global discourses to the interaction between friends:

> Special emphasis should be placed on the extreme *heterogeneity* of speech genres (oral and written). In fact, the category of speech genres should include short rejoinders of daily dialogue (and these are extremely varied depending on the subject matter, situation, and participants), everyday narration, writing (in all its various forms), the brief standard military command, the elaborate and detailed order, the fairly variegated repertoire of business

> documents (for the most part standard), and the diverse world of commentary (in the broad sense of the word: social, political). (p. 60, emphasis his)

In the researched classroom, we were all participants in the formation of this emerging genre. The new genre we were developing had to evolve from existing genres. But with which genres did this evolution begin? In which other genres did our first words seem to fit?

The students' frame of reference seemed to be formatted by their typical classroom communication genres. Their utterances in typical classes, and especially in their mathematics classes, had been and still were markedly passive. Gordon (1999) draws on the Bakhtinian notion of addressivity to describe such traditional discourse. In such discourse, the teacher "talks-at" students, ignoring their responses or expecting them to regurgite his or her own utterances. In short, the participant students in my research were accustomed to feeding their teachers what they wanted to hear. In this kind of discursive relationship, even genuine questions are taken as what Ainley (1987) and others call "guess-what's-in-my-mind" questions, where students are expected to read the teacher's thoughts and show their knowledge of the teacher's hidden answers.

Accordingly, in our conversations about language, the students seemed to be trying to tell me what they thought I wanted to hear, especially in the early stages of the research. Even the participant teacher, Mrs. Hill, seemed concerned for me to be hearing what I wanted to hear in terms of the research. She pointedly asked me at least three times, "Are you getting what you want?" I always answered with the affirmation that I wanted whatever would happen, that I did not want the result simply to confirm my expectations. Though I knew this to be the right answer for a researcher, I nevertheless recognized my anxiety about getting "good" results.

It is unclear to me to what extent Mrs. Hill asked me this question about results out of concern for my research, and to what extent she sensed my anxiety about how the unfolding events differed from my expectations. While I allow for such depth in her response to the research, I am aware that I hold a relatively narrow interpretation of the students' corporate response. I interpret student response to my prompts as an instance of their typical classroom culture – giving the teacher what he wants.

My frame of reference seemed significantly different from the students'. I entered this relationship as a researcher. I valued the articulation of varied perspectives. Using Gordon's (1999) terminology, I intended to "talk-with" these students, not "talk-at" them. I wanted to hear students' ideas – their disagreement with me and with each other. Not surprisingly, students were somewhat bewildered in this apparently contradictory space, which appeared to be new to them. It did not make sense for them to try to feed me what I wanted to hear, because I actually wanted to hear what they had to say when they were *not* saying what they thought I wanted to hear.

Even after four months of daily conversations about language, there seemed to be some confusion about the goal of our conversations. Indeed, different participants probably had different goals. Though there were some common goals, student goals were different from mine, and Mrs. Hill's goals were different still. Initially, she assumed my focus on language would have as its goal the fostering of students' understanding of the definitions of mathematics terminology. Some students shared this assumption through to the end of our engagement together. For example, near the end of the research, Kari summed up the value of our attention to language this way: “It helps show us ways, like help memorize the stuff that you learn and stuff.” She seemed to think that the goal of our conversations was to promote conventional perspectives. By contrast, I was more interested in drawing out students’ perspectives than in promoting conventional ones.

The tension between Kari’s motive and mine is an example of what Pickering (1995) identifies as the tension between *disciplinary* and *human agency*. In Chapter 5, I give an account of a more extensive stream of conversation in the researched classroom, a series of dialogues in which this tension became more clear. I wanted to draw out student utterances that revealed their human agency. But they were accustomed to regurgitating conventional understandings, a practice which accedes to disciplinary agency.

An Overview of My Prompts for Language Awareness

My structuring arrangements during the eighteen-week conversation with the students and Mrs. Hill shifted in response to both practical considerations and the students’ responses to me. Figure 4-1 provides an outline of this development.

Time Period	**How I Prompted CLA**
Weeks 1 – 3	waiting for video
Weeks 4 – 6	finding a place for CLA
First Threshold	*“Rigorous Pedagogy”*
Weeks 6 – 7	interjecting CLA
Second Threshold	*“Letter to Myself”*
Weeks 7 – 16	beginning each class with CLA
Weeks 17 – 19	*dénouement*

Figure 4-1. The evolving nature of my CLA prompts

For the last three weeks, which I call the *dénouement* in this chart, I was no longer a regular presence in the classroom. I had promised students access to transcripts that I might include in reports on the research. These three weeks afforded me the opportunity to bring our conversations to a close, with adequate time to transcribe the significant portions of our interaction. I also used this time to finish some less public conversations with students in interviews. My conversation with the class as a whole effectively ended at the close of the sixteenth week.

Waiting for Video (Weeks 1 – 3)

I co-taught with Mrs. Hill from the outset of the school term. We were both present every day, but we alternated the teaching. When students worked independently, we both circulated through the room and interacted with the students. For the first three weeks, I was waiting for the students' informed consent to fulfill ethical requirements (see Chapter 3). Though my prompts for language awareness were woven into my mathematics teaching in this time period, I was already becoming aware of problems associated with interpreting participant utterances.

I hesitated in these first three weeks to do anything significant relating to language awareness because I wanted to wait for consent to have recording equipment in the room. In retrospect, I think of this hesitation as a mistake, because the first days of the relationship set the tone for the whole term. In this time period, I did not make explicit discussion about language a clear part of the daily class agenda.

However, I did initiate some discussion about language practice during this three-week waiting period. I tried to find appropriate times to insert discussions about language each class. Though I rarely found a way to insert discussion about language on the days Mrs. Hill taught, on my days I was able to insert some prompts for attention to language. These prompts were, for the most part, integrated with discussion about mathematics.

Students did not seem to be aware that we were doing something out of the ordinary. Indeed, as Adler (2001) illustrates, all mathematics teaching involves the dilemma of transparency. In addition to using language transparently to communicate mathematics, teachers frequently direct some attention to language itself.

The students' responses to questions about language seemed similar to their responses to prompts about mathematics. The responses were minimalist; the students would say barely enough to satisfy me, their teacher. Ironically, though they appeared to be trying to satisfy me, I found the nature of these responses highly unsatisfactory. I had the sense, as I have often had in my prior mathematics teaching experiences, that these students avoided thinking. The extent of their cooperation was to do what was asked in its most elementary interpretation. Near the end of the term, some students in the class articulated this attitude during interviews – for example, when I asked Sita about the value of our discussions about language, she answered, "Well basically when you're in there you just want to get it done, learn it and know your stuff, and you don't really think about anything else. You know?" (week 15). Yes, I knew what she was saying. When I was her age, I often exhibited the same attitude.

Though most responses to my prompts seemed uninspired and minimalist, I was struck early in the conversation by the challenge I would face in interpreting students' utterances. For example, on the first Friday, I was leading a discussion about linear systems. I asked students why we might call these things linear systems. I was using the *we* voice intentionally, to draw attention to the voice of the discipline. Matt responded, saying that systems are things working together, and a few other students noted that the word *linear* referred to lines. One student said something like "It avoids confusion." (I could not recall who said it, because I was working from field notes, not recordings at this time.) This answer excited me, because this student was pointing to the need for shared meaning in a discourse. Using Bakhtin's (1953/1986) classification of meaning, the response suggested a need for proximity between the meaning of the neutral, conventional word and the personal, expressive meanings of "my word."

I asked this student to say more but the student remained silent. Instead, Joey disagreed with his classmate, saying that people use different words for the same thing all the time. It seemed that Joey too was aware of the flexibility of meaning to which Bakhtin points. As I understood the situation, their disagreement pointed at the tension that Bakhtin asserts is present in all conversations.

The following Monday, Mrs. Hill taught, concluding the class with a seatwork assignment. In the last minutes of the class, I asked for the students' attention and referred to our conversation the previous Friday in which we talked about meaning, albeit very briefly. After repeating their good reasons for using the name *linear systems,* I suggested that there might be other good names. I asked why *we* should call these things linear systems, strongly emphasizing the word *we*. I hoped to resurrect the disagreement between Joey and his classmate about conventional meaning. But the issue was not mentioned. Other students responded, saying the name makes sense, the name is not too complicated, and people cannot think of a better name.[5]

As I reflected on this brief stream of conversation, I thought about the "to avoid confusion" utterance. It seemed like such a good answer at the time. My initial thought was that I was finding success in terms of my research agenda because I was hearing the students' perspective on their mathematics classroom discourse, a major goal of the study. However, my personal revelry was quieted within minutes following the utterance as I reflected on it. Yes, it was a good answer, but how could I know that it was actually the student's perspective? Using Bakhtin's framework, I wondered: Was this the *student's* expressive word, the student's reflection of *my* expressive word, or the student's articulation of the neutral, conventional word?

This student may have heard another teacher say something like this in defence of conventionality. If this were the case, then the articulated perspective could still be the student's perspective, but borrowed. Alternatively, it could be an instance of the student simply saying what he or she thought I wanted to hear with the assumption that all teachers would share the same opinion on conventionality. Indeed, teachers are the face of enculturation in our society, the public's agent in introducing and enforcing conventionality.

Bakhtin (1974/1986) finds this phenomenon interesting too: "There is neither a first nor a last word and there are no limits to the dialogic context" (p. 170). Everything that is said has been said before. Despite this lack of originality in our speech, every utterance is unique because its meaning is contextual. Even if this student was repeating something heard from a previous teacher, it was still a new utterance because it had never been said in this new context.

What if another student had repeated the same words in the same conversation? The meaning would be different again. This possibility points to a problem with conversations involving multiple participants. I realized that every student utterance could be taken as a corporate expression, because students often see themselves as a corporate whole in juxtaposition to their teacher. When the student said, "to avoid confusion," this utterance did not need to be said again. Indeed, it could not be said again without meaning something else altogether. Thus, by making this utterance, this student cut off the possibility of our knowing whether any other student would have given the same or a similar response. It was possible for classmates to express agreement or affirmation with this student, something that some of them would do for other students in future conversation, but there was also the sense that silence implies consent.

With this problem in mind, how could I characterize this response? Was it this one student's response, or was it a shared response? Joey helped clarify the situation by disagreeing. By doing this, he broke the students' corporate sense, but the problem of interpretation persists. Once Joey disagreed, no one else needed to say anything in disagreement. While I wish I would have asked for a show of hands, to see who agreed with Joey and who agreed with the "to avoid confusion" utterance, I am aware that even this action could not have averted the inevitable difficulty of interpreting research participants' responses.

This problem, which permeates all the interpretations of participant utterances in this research, may be exacerbated in a conversation with thirty-plus participants, but it remains in more intimate clinical interviews. For example, in my interpretation of students responding to audio recordings of their mathematical conversations (Wagner, 2003b), I noted how my research participants followed my language form in the interview. With the consensual nature of conversation, it is impossible to say definitively that this participant *says* something in particular or students in general *say* something. In my reporting, I can only say something like "this participant *said*." This problem at the heart of all interpretation supports the necessity of framing my research as an investigation of possibility and of using narrative to frame my reports on the research. It is more accurate to give an account of interactions that have happened in my conversation with students than to make general statements about what students say or do. Readers can consider my accounts in their own frames of reference and evaluate the transferability of the situation to their particular contexts.

Finding a Place for CLA (Weeks 4 – 6)

Once I began recording class interaction, I became more explicit about my interest in language. In the first weeks, two issues relating to critical language awareness surfaced. First, to make language analysis effective, the teacher and students needed to take enough time to engage in it. Second, I found that my expectations were too high regarding participants' memory of their own utterances.

On the first day that I started recording the class sessions, I gave students some text to look at. Their initial response to this approach was promising, but our follow-up conversation revealed a general lack of interest or lack of depth in their evaluation of the discourse. I had hand-transcribed in real time a portion of class discussion a few days earlier, when no tape recordings were being made. Figure 4-2 shows the transcript I presented to them using an overhead projector. I introduced it saying how I had transcribed it and suggesting that any errors were likely due to my method. I asked them to read the transcript and tell me what they would see in it. The letter *T* refers to Mrs. Hill who was teaching at the time of the episode. The other letters, *A* through *D*, refer to different students. They are not the initial letters of the students' names. The dark horizontal lines represent conversation and time not included in the transcript.

T: Does anyone know what a graph of a quadratic looks like?
A: A curve?
T: [*draws a curve*:] ... like this?
A: I'm guessing.
T: Oh, you're guessing. That's good. It's [*a quadratic is*] not a straight line.

T: Write me a statement that describes what has to be there in an equation to make it a quadratic function. [*pause*] and then I'd like you to read them.
[*T then asks some people what they wrote, and they say they didn't write anything. Then T writes the question on the projector and gives them a little more time.*]
T: B?
B: I have no clue.
T: C?
C: a cannot equal zero.
T: Is that right? [*asking the class, and writing it down*] cuz what happens if a is zero?
[*some students answer this.*]
T: A?
A: There has to be an x and y value.
T: All right. Good. [*she writes it down*] D? What did you write?
D: I think there has to be an x-squared, like with a value in front.

T:	Does there have	to be a value in front?	[*simultaneous*]
D:		Like one even.	

[*Later T asks if* $y = \sqrt{x^2}$ *is a quadratic, having written the equation on the transparency and referring to it.*]
A: Doesn't the square root cancel the x-squared?

Figure 4-2. The first transcript shown to participant students

Though I later noticed that large transcripts like this one seemed to be overwhelming for the students with whom I was working, they responded favourably in this particular situation. Perhaps their relative interest was due to the novelty, this being the first transcript they would see. The following transcript is a representation of the dialogue we had about this transcript.

DW: So, what do you see in it, about the way people talk in mathematics class?

Matthew: Lots of questions.

DW: Pardon me?

Matthew: Lots of questions.

DW: Lots of questions? Where do the questions come from?

student: Teacher. [*long pause*]

DW: Should they always come from the teacher?

Joey: No.

DW: Do you think it's kind of strange that if you're the ones who are supposed to be learning that it's someone else asking the questions? Because you'd think that if you'd want to learn something you'd want to be asking questions – the other way around?

Matt: They're leading questions. To make you ... They're supposed to make us think.

DW: Leading questions are supposed to make you think? [*pause*] Anything else you notice there? Like I notice all kinds of things, but I'm not going to say what, because I want to hear what you notice. That's what I'm interested in in this study.

Signot: It's different from the language you usually use.

DW: It's different from the language you usually use? How is it different?

Signot: The way the teacher talks. [*pause*] It's just different.

DW: The way the teacher talks? Like, what's wrong with it? I mean, not *wrong*, but I mean what's different about it?

Signot: It's different.
DW: Different? Okay.
student: A foreign language.
DW: It's a foreign language. [*long pause*] Do you, in your other classes, not math classes, do people talk in the same way?
student: Yeah.
DW: Yeah? Because it can show us the difference between math and social studies, for example, if we notice the talking is different in the different places. And maybe even help us understand the math. Anything else you notice? [*long pause*] Okay.

Transcript 4-1. Students responding to the first transcript they would see

With this day's conversation, I was trying to draw out the students' observations. To do this, I acted as an amplifier by revoicing their utterances, and I explicitly told the students that I wanted to know what they themselves noticed. O'Connor and Michaels (1996) consider a variety of reasons a teacher may want to revoice student utterances. While I characterize the intention behind my revoicing as amplification, which O'Connor and Michaels call *rebroadcasting*, I recognize, as do O'Connor and Michaels, that such a move may have other effects, such as implying the legitimacy of the student's contribution and promoting a participant framework in which students talk to each other.

Looking back on this transcript and watching the video record of this series of utterances, I realize that I did not give the students sufficient time to respond. In the moment, I allowed long pauses relative to the pauses in their usual teacher–student dialogue, pauses up to six seconds long. Six seconds? When I analyze transcripts, I need to read and reread them. I need to dwell on them for more than six seconds. While six-second pauses seem laughably inadequate, in the mathematics classroom culture, where the agenda for mathematical content is formidable, it would be hard to convince students of the merits of dwelling for a long time on a particular transcript, especially when they have no experience analyzing transcripts. I can only wonder what would have happened if I had asked the students to consider quietly the transcript for a longer time or to talk about it with a neighbour before reporting back. With these kinds of prompts I would have surrendered control and allowed the very real danger of students avoiding the task and chatting about something else. I would not have wanted that kind of thing to happen in this first instance of transcript analysis, the episode that would set the tone for future conversations.

My reflective writing from later that day recorded my disappointment with the students' perception. As the conversation between the students and me continued past the above excerpt, students talked animatedly about how they could not remember the incident in the transcript I showed them. As Mrs. Hill approached the front of the room to take over the class, she asked the students, "Do you remember that conversation?" The students surprised me by indicating that they did not remember it. I asked them if they could discern which students the letters *A*, *B*, *C* and *D* indicated. Even the students who made the quoted utterances could not remember the dialogue. In my reflections, as recorded in field notes, their forgetfulness eclipsed our discussion about particularities of the language:

> It feels like I didn't learn much about what they see in their discourse. It seems that there is value in giving them a taste of what a transcript looks like, and the realization of how quickly they distance themselves from their dialogue – because they can't even remember the conversation, let alone who said what.

This phenomenon of forgetfulness was significant because one of the reasons for my interest in having them look at their own discourse was that they would have access to the intentions behind their utterances (see Wagner, 2003b). I realized now that I could not expect to experience this sense of greater proximity or access to their intentions. Mrs. Hill's way of phrasing her question also suggested distance from the episode, even though she herself was a participant in the transcripted dialogue. She used the distal pointer *that* instead of the proximal pointer *this* when she asked, "Do you remember *that* conversation?"

Despite the inadequate time I gave the class and despite their inability to remember the situation, students did notice features of the language in the transcript. Matthew pointed out the density of questions and Signot noticed that the language was somehow inexplicitly different from everyday life. For upcoming discussions, I decided to pursue Matthew's attention to questions.

A few days later I showed the class a short video clip of a class discussion that I had led. I asked the students to pay particular attention to the questions in the clip. To my surprise, the students did not even watch the whole video clip, despite its brevity – five minutes. Many turned away and started chatting with each other halfway through it. Though they were not literally silent in response to my prompt and the video clip, their response was effectively quiet. They ignored my prompt. After playing the video clip, I used the overhead projector to display a list of the questions asked in the video clip, and asked the students what they noticed. The only thing they pointed out was which students had asked the most questions. It was like a popularity contest. Some students did not appear to notice that the projected list represented only the questions.

Interjecting (Weeks 6 and 7)

Over these first few weeks of trying to draw attention to language, I was disappointed with the students' response to my prompts, but I was more disappointed with myself. I felt as though I was giving them too few prompts and that these prompts were not good enough. I found myself too easily distracted from my plans to start conversations about language. In order to encourage myself to be more deliberate about prompting attention to language, I wrote a short reflective paper called "Rigorous pedagogy" and discussed it with Mrs. Hill. The following excerpt reflects my negative regard for what I had done up to that point. (I will be referring to excerpts here, but the paper is presented in its entirety in Appendix B.)

> I think I've come to a crossroads in the research. I've noticed that I have been feeling uncomfortable with my initial approach to language awareness in the mathematics classroom. I have become less and less willing to lead the class in discussions about transcripts or about features of language. Now I think I know why.

Reflecting on my own self-criticism, I realize that typical classroom discourse makes certain kinds of discussion easy and others difficult. The introduction of any new genre is likely to be met with problems. The genre could not be expected to have a natural fit anywhere, because it was new and not yet part of the discourse.

In "Rigorous pedagogy," I said that I thought the reason I was having difficulty finding suitable times for language awareness prompts was that my orientation was geared more to pedagogy than to research. I saw in Mrs. Hill a strong pedagogical orientation, which is the kind of orientation I would hope for in any teacher. Though I considered my orientation to pedagogy healthy, I had a competing orientation – research. Though I was aware of other researchers who had struggled with conflicting orientations (e.g. Ainley, 1999; Phillips, 2002) and though I saw my multiple roles in this classroom as instructive in my analyses and interpretations, the problem still occupied my attention. When considering my experience with this class in juxtaposition with my experience of two classes in my master's research (Wagner, 2002a), I found that it was much harder to intervene with a group of students I saw every day, and with whom I had a teacher role. In "Rigorous pedagogy," I reflected that

> the pedagogue in me has resisted the apparent interruption of our mathematical journey to consider something else, even though this "something else" has potential for bringing richness to the mathematical journey.

At various times I had reflected in my field notes on the challenges of researching in this particular class, in which I was *a* teacher but not *the* teacher in charge. I was responsible to students and their parents, but I was also responsible to Mrs. Hill's leadership, even though it would have been impossible for her to fully understand my agenda for language awareness. She had some understanding of it, but she was not as consumed with the research as I was, nor should she have been. It was in the fifteenth week (the fourth from last week) of our work together that she said to me, "Now I get it." In that interview, I for the first time had the sense that she understood what I was trying to do with language awareness. In retrospect, I see that my high expectations for her were unreasonable. My own sense of the value of language awareness took years to develop. Though she would have the benefit of seeing me try to enact my vision for awareness, I ought to have expected her to need time to develop an understanding of my agenda.

In my reflections during the fourth week, I compared the responsibility I owed my participant students to the very different relationship in my master's research (Wagner, 2002a), in which I had entered the classroom as an outsider and in which my interventions had been brief and finite. This excerpt from field notes illustrates my growing awareness of the challenges associated with extended involvement in a researched classroom:

> I think that there is an interesting tension relating to trust. Students probably trust me more than they would an outside researcher. They trust my judgment because they see me seeing them pedagogically. But, on the other hand, they may be more tolerant of an outsider, because there is an awareness that the odd things he/she does are finite – he/she will be gone tomorrow. With me, when I do anything they consider odd, they may be wondering whether every day will be like this, and with this fear, they resist.

I also recall noticing the stress faced by Mrs. Hill. She worried about finishing the required curriculum and she frequently articulated her worry. This is an appropriate concern for a teacher; in my high school teaching experiences, I have heard the same concern expressed by colleagues numerous times. If my research were to have involved short, finite interactions with students, Mrs. Hill would have been able to mop up any "mess" after I left. However, at times it seemed to me that because I was there every day, she tried to avert any possible mess by resisting interactions that she considered unpredictable, which would include any prompts for which she could not see the pedagogical value. I think that this resistance was subconscious on Mrs. Hill's part, because she generally welcomed my presence in the classroom and repeatedly told me I could do whatever I wanted with the students. In future weeks, when my language awareness prompts would become more predictable, she was silent during my ensuing conversations with our students. She developed a routine of reading her e-mails in that time frame each day.

My frustration continued until week seven of our interaction, when I wrote the following reflection on the Wednesday:

> I worked so hard to get ready for a little chat about pronouns and agency today. I didn't have that chat with the class. My preparation meant hours of analysis of two lessons, and preparation of charts. It is the story of my research so far – having grand plans to talk about language and then not following the plans.
>
> Why did I not today? Here are some realities that confronted me today. Eight students were away last period. These people needed to catch up to the others. I had to finish the topic Cheryl [Mrs. Hill] started that day and then start and finish another topic for today. In our conversation at the end of last week, Cheryl only said she would want me to finish the lesson she started. This would have given me more time. Also, before class, Cheryl told me we are a little behind because of missing the double period (yesterday's PD day). Then, halfway through the class, Cheryl told me that this class was selected to be "invited" to watch volleyball on Friday. I thought I could have the little chat about language at the end of the class, when everyone is chatting about non-math. However, I saw about 10 students working diligently on their math, students who missed last day some of them. How can I interrupt their time to work on what I asked them to work on, I wonder? I guess this is always the problem for teachers.
>
> On my way home I reflected: perhaps I need to change my tack.

This reflection prompted me to write a letter of encouragement to myself, a reminder of the things I wanted to accomplish with these students. After this letter, I began a routine in which I would begin each class with a few minutes of attention to language. I hoped that these brief daily interactions would resurface in subsequent discussions about mathematics, but they never did in any significant way.

While writing this letter to myself I recalled my passion for challenging routine practice by introducing risky alternative possibilities. I wrote this letter early in the morning on the day following the reflection quoted above. I reread the letter just before class began to bolster my confidence. Yes, I was feeling like a failure. This is the beginning of the letter:

> Take courage, Dave. Your desire to see a more critical, more self-aware mathematics learning experience is important. Though the world around you tends to be content doing the things that are normally done, you are disturbed. Do not let the complacency around you deter you from your significant endeavour. You have been inspired by the tenacity of the people of southern Africa, who have shed, to some extent, their colonial masters, and who continue to recognize the need to think about what they are doing when teaching – even when teaching mathematics.
>
> Take courage, Dave. Ask Cheryl to give you the first two to three minutes of every class to talk about classroom discourse with the students. You can assure her by welcoming her to say "time's up," and even asking her to be sure to say it, so that you can concentrate on interacting with the students, albeit briefly.
>
> Take courage, Dave. Follow the example of Carolyn [my wife] who wrote [in the community newspaper she edits] an editorial that she knew would disturb/perturb the community's complacent readers. You know that her courage to write this editorial is connected to your own encouragement. You told her

> that a disturbing/perturbing editorial would be well received,
> despite its "abnormality." Follow your own advice, Dave.

Perhaps it is significant that I wrote this letter on my birthday. It was a new beginning, a time to reflect on the past and begin afresh.

Beginning Each Class with CLA (Weeks 7 – 16)

I did follow the advice in the letter to myself. I asked Mrs. Hill for the first few minutes of each class regardless of who would be teaching – she or I. This approach to language awareness appeared to be the most generative. The three conversations that I will be discussing in detail in the next three chapters all began in our short language awareness talks at the beginning of each class. Though these three significant streams of conversation arose out of this time of regular language awareness prompts, they cannot be attributed directly to the nature of these interactions. There was at least one other significant factor: These generative exchanges appeared close to the end of the term, after students had had more exposure to explicit prompts regarding language.

Though this final period of the conversation was the most generative, I still characterize the students' participation through this time as passive resistance. The transcripts I include in this and the next three chapters can be deceptive. In transcripts, we report utterances, not silence. These three streams of conversation exemplify possibilities. They do not typify what happened in the daily interactions I had with students in this classroom. Rather, these dialogues were typically composed of various forms of silence. The next part of this chapter focuses on these silences.

Silences

> Understanding comes neither from a lot of talking nor from busy
> listening around. Only he who already understands is able to listen.

> Another existential possibility of discourse has the same existential foundation, *keeping silent*. In talking with one another the person who is silent can "let something be understood." (Heidegger, 1927/2003, p. 258, emphasis his)

Heidegger points out the communicative effect of remaining silent. In my conversation with students about our language practice in the classroom, they were often silent in response to my questions. What does this mean? Silences are difficult utterances to interpret.

Taking the utterance as the basic unit of language for analysis in this research, it is necessary to extend Bakhtin's notion of the utterance so that it can include silence. When there is a communicative effect associated with a particular silence, I am calling it an utterance. Doing this, I am aware of my divergence from Bakhtin's (1953/1986) delimitation of an utterance – the oral or written text that begins after another person's utterance and ends when someone else speaks or writes in response. Bakhtin considers silence to be a response within certain genres, but he claims that "this is, so to speak, responsive understanding with a delayed reaction. Sooner or later what is heard and actively understood will find its response in the subsequent speech or behaviour of the listener" (p. 69). I counter that some, but not all, silences are taken as full responses in themselves.

Consider the following situation. I ask my daughter, "What are you doing now?" She hears me and knows that I mean something like, "You need to start your homework now." She responds with silence. I wait patiently (at least I consider myself patient) and then reply, "Okay, if you sit there silently, doing nothing, you will not have much time to read before bed." Clearly, the two things I said – "What are you doing?" and "Okay, [...]

you will not have much time" – are separate utterances. My second utterance is in response to my daughter. This is not the same thing as a pause in my utterance. Bakhtin's (1953/1986) characterization of pauses takes no account of another person's intervening response; it is part of the speaker's intention: "Any pause that is grammatical, calculated, or interpreted is possible only within the speech of a single speaker, i.e., within a single utterance" (p. 74).

I am suggesting that silence can be an utterance, but not all silences are utterances. As Bakhtin (1953/1986) shows, some are mere pauses. There are different kinds of silence. Heidegger (1927/2003) says that silence only speaks when it is in a context of prior speaking: "He who never says anything is also unable to keep silent at a given moment. Authentic silence is possible only in genuine discourse" (p. 258). An angry refusal to speak to someone is an example of authentic silence. Clearly, the silence speaks. And the silence speaks clearly. Considering the communicative significance of silence, the participant students' silence needs to be interpreted in the context of their general silence relating to everything in the classroom. Because they were generally silent about other things, their silence in response to my prompts for language awareness might not be considered significant. They were not withholding comment. They were just accustomed to being voiceless, dumb. And they were continuing in that role.

This kind of continued silence is significant in a different way. Neumann (1997) tells stories about her mother's silence. Her mother said nothing about some of the most important experiences in her life; she did not speak about the Holocaust and she did not speak about her own hard work to support her husband's career. Neumann shows how sustained silences uphold spoken discourses and how spoken discourses silence other possible discourses. I know my own silences better than I know Neumann's mother's silences, so I will refer briefly to these to demonstrate the mutually sustaining relationship between silence and spoken utterances in any discourse.

There are many silences I keep. Sometimes I am aware of particular silences, but the nature of silence is that it sustains ignorance. It is because of my silence in particular areas that I forget about these things. I ignore them. One silence I keep is to refrain from asking my father questions about his adolescent experiences in Poland and Germany during World War II. For a time I tried to get him to share his experiences, but he did not want to tell, partly because he wished to forget and partly because he could not tell things that would not be understood. Sometimes he tries to tell, and he has explained why he cannot tell. My silence sustains his silence. My silence permits him to remain silent.

I know the challenges of telling a story that is so far removed from my audience's experience that there is no basis for understanding. This problem is an integral part of Heidegger's (1927/2003) description of the way we attune to phenomena and understand them: "Only he who already understands is able to listen" (p. 258). I do not tell my Canadian friends some of my most significant experiences in Swaziland, because I know they will not understand. Yes, they would come to *an* understanding, but that understanding would be based on predispositions that I consider inappropriate pretexts for the experiences that I would want to share. This kind of silence is self-censored.

In addition to these chosen silences, there are repressions that sustain silences. There are certain kinds of experiences and understandings that are not considered appropriate in academic discourses. Neumann (1997) warns of the need for researchers to be aware of their own silences in their writing. She suggests that these silences sustain the stories that are told and she promotes the excavation of these silences to help us understand spoken and written discourses. In my first two chapters, I gave accounts of the background I brought to my research. There is more. I said little or nothing about my relationships with my parents and siblings, my feelings about these relationships, events from my relationships with friends while growing up, my relationships with my wife and children, and my relationship with my God. Ignoring this background promotes a certain kind of account of the research. It is not that the promoted account is inappropriate, but rather that it is only one perspective on the situation. I am aware of the unwritten boundaries in academic discourse, barriers that keep these kinds of formative relationships and experiences silent. Along with my complicity to live within the discourses, and to respect their boundaries to some extent, I am aware of the difficulty with which one pushes these boundaries. I am not sure how I would begin to break through these barriers.[6]

To summarize, there are silences that speak by being juxtaposed with utterances that might have been offered. There are silences that we choose for ourselves to protect our interpretations of our experiences, or to avoid painful memories. And there are repressive silences that are endemic to particular discourses. In light of these types of silences, it seems that a proper analysis of any discourse should include a consideration of the silences within it. Silence has become an important theme in my interpretations of the researched classroom. I will return in the final chapter to examine the silences in this thesis.

My conversation with the students and teacher about language in mathematics became its own discourse, which was situated within a greater discourse of mathematics classrooms. The students' silences and spoken utterances were situated within both of these discourses. In this research, I am trying to address two silences in the greater discourse of mathematics education. First, by introducing critical language awareness to this one classroom and by writing accounts of the result, I address the general silence about language in mathematics classrooms. It is not a complete silence. There is some scholarship that addresses this feature of the discursive practice, but I assert that more attention needs to be drawn to it. Second, by trying to draw out students' interpretations of their own language practices, I address a general silence in education scholarship – students' interpretations are rarely considered. Typically, researchers do the analysis, while teachers are sometimes drawn into interpretative conversations.

In the following sections, I interpret silences within the researched conversation I had with students. This interpretation of silence is highly speculative. How can one analyze something that has no external representation? While it is appropriate to question the validity of interpretations of silence, it may help to remember that the interpretation of *any* utterance is highly speculative. Furthermore, the properties of mathematics itself are similar to those of silence. Mathematics is the study of things that have no external representation (cf. Balint, 1968). The difficulty of representing and interpreting silence will be taken up again in the final chapter of this dissertation.

Student Silences

I have been saying that the students were generally silent in response to my prompts about language. Considering the various reasons one might have for keeping silent, it should not be surprising that these students kept their silence in more than one way. Their shallow responses were like silence to me. Their responses to my requests for interviews formed another kind of silence. Some students did not agree to be interviewed and others agreed but did not appear for their interviews. I will suggest that both of these kinds of silences, in addition to their literal silence in response to many of my prompts, were closely related to the general silence regarding language in the mathematics classroom discourse, the silence I want to address.

The first kind of silence that I represent is much like a politician's silence. It is when a person is not silent *per se*, but says relatively meaningless things to fill the gaping void that would be obvious if the speaker maintained absolute silence. The following transcript of an entire interview with Kari exemplifies this kind of silence. I began the interview by asking her what her goals were for this class, a question with which I began many interviews. This was in the sixteenth week of my conversation with the students in her class.

DW: [What are your goals for this class?]

Kari: Um, I wanted to learn the course and pass and go into Pure Math 30.

DW: Okay, same as almost everyone else. Okay, now when we talk about language and stuff, that stuff that we do, do you see that as contributing towards you achieving your goal in any way, or not?

Kari: Hm, it helps occasionally but sometimes I don't see the connection.

DW: And when does it help? Or, how does it help or whatever?

Kari: It helps show us ways, like help memorize the stuff that you learn and stuff.

DW: Can you give an example?

Kari: Um, not right now.

DW: No? If you think of one, please tell me, because you're the first person who's said that it helped memorize things.

Kari: Oh, okay.

DW: Someone said that it helped them remember, but that's different from memorizing, right?

Kari: Hm-hm [yes].

DW: Okay. Can you think of a time that you were doing math, like homework or a test or something, and then you noticed something about language, something that we had talked about at one time?

Kari: Hm, no.

DW: Do you know that it hasn't happened, or is it just that you can't think of a time?

Kari: I can't think of a time. I don't. [*long pause*]

DW: Yeah, that's fine. Do you enjoy talking about the language and stuff or do you find it sort of.

Kari: Yeah. I do, because like it's a break from math and stuff, and some of it's interesting.

DW: Like, what did you find the most interesting?

Kari: Like the stuff where we interact. Like where we got up and did that Venn diagram. That was an interesting way of learning about Venn diagrams.

DW: Do you have anything else to say about this language stuff?

Kari: No.

Transcript 4-2. An interview with Kari

Throughout this interview, Kari was smiling pleasantly. It seemed that she wanted to be helpful, because she seemed happy to be interviewed. Some of her classmates did not want to be interviewed, and she was aware of their polite refusals. Furthermore, she spent a few seconds thinking about most questions, time that she filled with sounds like "Um" and "Hm." But she did not seem to have anything to say. She gave me concise answers to my questions, answers that suggest she had nothing to say. I consider this to be a kind of silence – empty words and words that suggest emptiness.

Consider Kari's utterance "it helps occasionally but sometimes I don't see the connection," for example. I asked her if our discussions about language helped her achieve her goals. She effectively avoided the question with her answer, "it helps occasionally" and with her follow-up, in which she said she had no examples. If she were to have answered, "No, it has not been helpful," she would have been inviting more questions, because she would be speaking with high modality, a high degree of certainty. Such an answer would have suggested that she had strong feelings about our language conversations. By contrast, her answer suggests relatively weak feelings. She softened her agreement with the position she would have expected me to hold, using the hedging word *occasionally* to reduce her commitment to her assertion that the conversations helped her. When I asked her to clarify with an example of what she meant by the word *occasionally*, she had none. This interchange suggests that she was not using the word literally. Rather she was using it to hedge her commitment to the topic of our conversation.

In my next series of questions, I again asked her to give an example of a time she noticed language features while doing mathematics. Her answer, like before, suggests that she agreed with the position she would have expected me to take, but her response was offered with low commitment. When I followed up with a request for an example, she adjusted her approach from the first time I made such a request. When she said *occasionally*, I asked her to clarify what she meant by the word. This time she just said no. Again, I pursued her negation by asking for clarification. Her response to my pursuit ended with the silent treatment, with literal silence. She began a sentence and did not finish it. With this long silence at the end of her utterance, she was telling me that she did not want to be asked about details. I persisted out of a sense of duty to my research agenda, but Kari really controlled the interview with her silences.

Other students gave similar silences in their interviews. Some students said they did not want to be interviewed, but I failed to record these refusals because at the time I did not realize the significance of these silences. Some students agreed to be interviewed but did not show up at the agreed-upon times. Joey and Gary, who seemed to be more interested about our discussions about language than any of their classmates, did not come to our first two scheduled interviews. Signot also seemed interested in our discussions about language and about mathematics. Like Joey and Gary, he did not appear for his first interview though he had agreed to come. The same thing happened with Brandon and Neeta. Matt and Matthew agreed to come for an interview, but only Matthew appeared, so we rescheduled to accommodate Matt, who apologized and assured us that he would be present the next time. He did not appear then either. In all these instances, the students had assured me that they would come for their respective interviews. We had negotiated our schedules to find appropriate times, but they did not appear. The only students to appear for a first interview at the agreed upon time were Matthew and, in the last week of the research, Arwa and Tharshini. In every instance, students had plausible excuses for not coming, but the pattern was quite clear. For me these lost interviews were powerful silences. Each time, I sat quietly in a literal silence, waiting in an empty room for the students to appear during our agreed-upon times.

It was because of the students' general interview-avoidance that I adjusted my strategy to catch them in interviews. I invited individuals and pairs of students into the hallway for impromptu interviews at convenient times – after they finished writing a test or when they were supposed to be doing seatwork but were not actually doing any, for example. The above interview with Kari was one such impromptu interview. The dynamic in such an interview is significantly different from an interview in which participants agree in advance to come and actually come. I suggest that participants are less likely to follow the interviewer's formatting in an impromptu interview. They do not enter such interviews with a predisposition to be compliant.

In addition to their interview-avoiding silences, when I prompted the class as a whole to discuss their language practice, the students avoided my prompts in different ways yet. They could not be physically absent from the conversation and they did not feel the need to fill empty moments with shallow answers, as some of them did in interviews. With approximately thirty participants in a conversation, each person can have the sense that someone else will say something, and so avoid personal responsibility. When all the students took this approach, literal silence was the result.

I asked the students about this silence in my final days with the class. I showed the students a video clip of their response to one of my language awareness prompts, then at the beginning of the next class I referred to that video and told them that I was calling their response in such situations "passive resistance." I said that their silence could mean many different things; it need not suggest resistance. The following transcript is a representation of the ensuing interaction. (The student referred to as "girl" was outside the camera range, and so her identity is not known.)

DW: Can you tell me to what degree you felt like you were cooperating and doing what I asked you to do? [*5 second pause*] When we talked about language things? [*3 second pause*]

girl: I think half of us were being lazy and half of us just didn't want to say anything.

DW: What did you say, again?

girl: Half of us were being lazy and half of us just didn't want to say anything.

DW: Which is kind of being lazy, kind of thing.

girl: Yeah.

DW: Yeah.

girl: Half of us.

DW: Do you guys agree? [*gesturing to the whole class*] She's judging you, just like I was judging you. And I realized I can't do this, I should *ask* actually. [*motioning to Matt, whose hand is raised*] Yup?

Matt: Well, one thing is, I don't think everyone just sat there. I bet you almost everyone thought. But at some times, you know, you just don't notice anything.

DW: Yeah, like the thing in the video.

Matt: Yeah, like some people probably tried but just didn't notice anything.

Kyle: What was the question you asked for us to answer?

DW: [*recounting the video clip and saying what prompt was given to the students – to attend to the* we *voice*]

Jessye: I didn't notice anything.

DW: Yeah, some of you probably didn't. But if you *would* have noticed, would you have said? That's what I'm basically asking.

Signot: Yes.

Brandon: Sure.

Arwa: I think some people just don't say, because we think we'll sound stupid.

DW: Okay, so sometimes if I asked something, you might not say something because you are worried about how you'd sound?

Arwa: Yeah.

DW: Do you? Or are you talking about others?

Arwa: Myself, I guess.

DW: So that happened to you sometimes.

Arwa: Yeah.

DW: Did it happen to anyone else? [*gesturing to the whole class*] Where you thought of something, but you didn't really want to say anything because you didn't know how it would sound? Put up your hand if it happened to you. [*no* *one put up a hand*]

girl: I just didn't because I didn't feel like it.

[*extended silence*]

Transcript 4-3. A discussion about silence

The girl outside camera range broke the literal silence with which her classmates responded to my question. She broke it with a different kind of silencing. In her answer to my query about their tendency to not answer my questions, she seemed to be speaking as a representative of the un-lazy half of the class. Her use of the word *just* masks the reasoning behind the action that is vaguely described – "[we] just didn't want to say anything." (In Chapter 8, I will report on my conversation with students about the use of the word *just* in mathematical communication. The effect is similar here.)

I did not let this girl speak in behalf of the entire class when I asked for other responses. From this point forward, the students were actively responsive. Matt said that their silence could mean that they did not notice anything. As I stated earlier in this chapter, I had given insufficient time for these students to analyze classroom texts carefully. I suggest that the kind of silence Matt was describing relates to a general lack of resources, of which time is only one. It is only when we have words and prior relevant experience that we can find meaning in any given situation.

This necessity for predisposition is described by Heidegger (1927/2003), who suggests that phenomena only present themselves within the framework of our fore-conception: "The interpretation of something as something is essentially grounded in fore-having, fore-sight, and fore-conception" (p. 244). The understandings that people come to are more a representation of the possibilities to which they were predisposed than they are of the phenomenon itself: “In the projecting of understanding, beings are disclosed in their possibility” (p. 244). The complexity of interpretation is also at the root of the disagreements between Fairclough and Stubbs, which I outlined in Chapter 2. Stubbs resists the predispositions critical discourse analysts bring to their texts, but Fairclough seems to recognize that predisposition is unavoidable.

Not only did the brief time frames allowed for the students’ interpretations prevent them from responding, but I believe that their silent responses to my prompts were also due to their lack of sufficient experience talking about language practices. They were not sufficiently predisposed to notice features of language in the given texts. Though this deficit of experience could seem on the surface to be a problem, it allowed them to experience the texts differently from the way I would experience texts, with my predispositions formatting my perception.

Shortly after Matt made his argument, Jessye articulated support for it. She said that her silence in the videotaped episode was due to her not noticing anything. I interpret her utterance as a general agreement with Matt, though it appears that she was giving an example from a particular incident. Given the apparent lack of memory these students demonstrated to me in previous conversations, I doubt that she could have remembered what she noticed or did not notice in an episode from long past. I think she was projecting her present attitude onto the video image of herself. Because she considered herself compliant, she interpreted that her silence suggested she had not noticed anything that would answer the question I had asked of her and her classmates.

After Matt’s account of his and his classmates’ silence, Arwa suggested yet another reason for silence. She said that she withheld her comments because of her fears.

None of her classmates admitted sharing this fear, but the fact that they did not raise their hands in other straw votes proved straw votes to be an inadequate measure of their thoughts. It could well be that many of Arwa's classmates shared her fears at times. Perhaps they were not raising their hands because of the same kind of fear of embarrassment. It is possible that my way of interacting with these students gave them good reason to fear being maligned, but I believe that Arwa's fear, which may or may not have been shared by her classmates, was related more to repressive silences in the general mathematics classroom discourse.

I think it is significant that Arwa, who expressed the possibility of fear, is a girl. Most of the conversation about language was carried by the boys in the class. As Walkerdine (1988) illustrates, boys are tacitly encouraged to engage in *rule-challenging*. My conversations about language were about rule-challenging. They were about finding alternative possibilities. By contrast, Walkerdine asserts that girls' successes are typically attributed "to rule-following and rote-learning" (p. 209), which my language prompts were resisting.

With this class, I noticed that once particular students said one thing in our conversations about language, they were far more apt to speak again in the future. Shortly before this conversation about the class' cooperation, Arwa had become involved in spirited interaction about the way she and her classmates directed their attention at words and symbols. (She and her friend Tharshini were the protagonists in this stream of conversation reported in Chapter 7.) I believe that she had the courage to admit her prior fears only because her fears had been recently alleviated.

I suggest that each of these silences that formed part of the students' response to my prompts for critical language awareness is related to the silences endemic in everyday mathematics classroom discourse. Their silence in whole-class discussion can be attributed to their not being predisposed to see the features of their discourse that I hoped for them to see, namely the lexico-grammatical features that I was predisposed to see. Also, any fear of speaking they had can be seen to relate to repressions that are typically engendered in mathematics classroom discourse. Gender bias is one such repression.

The apathy expressed by the unnamed girl in our conversation about the students' sense of their own cooperation, which resembled the apathy expressed by others in their resistance to interviews, can also be related to the general mathematics learning discourse. At first, the students simply did not see the relevance of language in their mathematics class. Various students articulated this perception throughout the months of our conversation. I also find this sentiment when I tell friends and acquaintances about this research project. "What does language have to do with mathematics?" is the typical response I get. That people in general, and these students in particular, do not and did not see the relevance of language in mathematics is very much a part of our wider culture's beliefs about mathematics.

However, some of the students in this research did come to realize the significance of language to their mathematics learning. Most of these students only came to this realization late in the conversation. In the next three chapters, I will give accounts of some of these students' observations. With my prompting, these students have been able to address some of the language-related silences in mathematics learning discourses.

To clarify, I have been saying that the silences in my research are *related to* the silences endemic in the general discourse. I have not said, and I will not say, that these students' silences were entirely caused by the silences in the discourse. Students can be silent because of distractions related to situations outside school. And students can be silenced by actions taken by their particular teachers. I recognize that I may have inadvertently silenced some students in this conversation.

Teacher–Researcher Silences

In the above overview of my approaches to prompting conversation about language with these students, I have referred to my frustration over my *own* silences in addition to the students' silences. I found myself not introducing language-related tasks that I had planned to introduce. I reiterate an excerpt from my reflective paper "Rigorous pedagogy":

> [T]he pedagogue in me has resisted the apparent interruption of our mathematical journey to consider something else, even though this "something else" has potential for bringing richness to the mathematical journey.

Who is this "pedagogue" that I was referring to? As a mathematics teacher, I represented the province, whose curriculum I was obliged to teach, and a large culture of mathematics educators. It is no surprise to me now that I felt compelled to follow the traditions in this profession. Included in this tradition are its silences.

Though my research agenda was to address some of these silences, I found that the silences in the tradition eclipsed my agenda. All the people with whom I was working – the students, their parents, Mrs. Hill and her department head – had a preconception of what mathematics learning should include. Even though they had agreed to welcome something new, they resisted this new thing to various extents. In turn, I sensed this resistance and feared pushing too far.

There is one other silencing force that I felt strongly throughout my interaction with the students in this class – the university's ethics protocol. I suggest that this force silenced me in some ways and also to some extent silenced the students through me. By virtue of the university's requirement for informed consent, I needed to explain to the students and their parents at the outset of our relationship my particular interest in this research. I was bound to tell them that our attention to language would be a departure from normal practice.

With this information, I drew attention to the oddness of our conversations about language. Other oddities bolstered this reminder of oddness – the video camera recording every class and the presence of two teachers, both features uncommon in mathematics classrooms. How could these students have been expected to take conversation about language as normal with these cues to the oddness of the situation?

I am not arguing against the presence of ethical protocols. I support the concern for the research participants promoted by the protocol. Indeed, I expected that some of the protocol's requirements would help me with interpretation. The protocol and its tradition expect researchers to share with participants transcripts and interpretations that might be publicly presented. Two of the three streams of conversation that I will present as positive examples of the possibilities associated with critical language awareness were prompted by interpretative papers that I wrote and shared with the students in the class (see Chapters 7 and 8).

Rather, I am drawing attention to the difference ethical protocols make in classroom research. With the requirement for informed consent, it is impossible to give students the sense that they are part of a "normal" class. With this interference, I question the extrapolation of any classroom research to form generalizations about mathematics learning when the research involves participants' informed consent. In my investigation, the ethical clearance not only delayed the introduction of video recording and my more explicit language-related tasks, but it also continued to draw attention to the oddity of the situation, and thus continued to support the students' perception that mathematics and language are not related to each other. The ethics protocol supported the typical silences that are associated with this perception.

Meeting Silence

In this chapter, I described the students' silences in response to my attempts to promote critical awareness of their language practice. Sometimes silence is the best response to silence – for example, when we sit silently with a friend who needs presence but not words. Some of my silences as a teacher and researcher were of this type – respectful reflections of the silences desired by my conversation partners. But it is not always helpful to grant people their silences. The next three chapters describe times when I drew the participant students out of their silence, and the final three chapters address silences in mathematics education scholarship.

Chapter 5 – Agency in Mathematics

> You shouldn't use any voice, you should use the general voice. I've termed it the general voice because I'm cool and I can make my own terms.

These are the words of Joey as he reflects on his use of the *I* voice in his mathematics class. This proclamation, together with its context, illustrates a possibility opened up when mathematics students become more aware of their language practice.

The conversational strand I describe in this chapter should not be taken as typical. As stated in Chapter 4, most of the students' responses to my critical language awareness prompts were not so generative. Silence was a typical response – silence in one form or another. The conversation with students presented here shows a possibility that can be opened up by critical attention to language. I further show how this strand of the extended conversation relates to what Pickering (1995) calls the "dance of agency."

In Chapter 3, I noted that Valero and Vithal (1998) encourage researchers to expect significant turning points in research to be prompted by disruptions. For me, it was a disruption when Joey and his classmates made their assertions about the *I* voice, when at the same time the students were using the *I* voice extensively during conversation about mathematics. I was both disrupted and disturbed. I still feel unquiet when I recall and recount this development.

Joey's proclamation about the *general voice*, with which I began this chapter, has its roots in one of these disruptions. I led the students in a conversation about human initiative in mathematics. We looked at voice in their utterances from the standpoint of who has agency in the discourse, who has control over the way the mathematics is done and expressed. In particular, their initial interpretations of their personal pronoun use were very different from mine, but we all learned something through this tension. After I present some highlights from this dialogue, I will show how the tension in our conversation about their language practice was similar to tensions inherent both in mathematics and in the forms of language commonly used to express mathematics. I will then direct attention back to this particular classroom with these tensions in mind.

The Story – Directing Student Attention to Voice

Our conversation about voice began early in our semester together, before I had begun recording class interaction. In this account, I divide my interaction with students into three parts. First, I describe how I introduced the students to the terms *agency* and *voice*. Second, I explore how the students used this awareness in their responses to particular student utterances. Third, I describe a reflective conversation in which the students and I discussed one student's analysis of school mathematics texts.

Introducing *Agency*

To familiarize students with the concept of agency and to exercise their ability to locate agency in utterances, I read them a newspaper article about a popular Canadian singer, Shania Twain ("Twain's World Tour," 2003). This was in the second week of class. I read one sentence at a time and asked the class to identify who, if anyone, was said to be making things happen. Who had "agency"?[7] The following excerpts from my field notes describe my introduction of the activity and my intentions:

> At the end of the class I said that we would do something decidedly unmathematical. There were three minutes left when I said [...] I wanted to talk about agency, which, I said, refers to who makes things happen in a sentence. Who is the agent, the actor? I didn't

> write anything anywhere. This was all oral. I said that I would ask them in the coming weeks about agency in mathematics, but that I wanted to give examples outside of mathematics so that I don't reveal my opinions about agency in mathematics. I will not want them to regurgitate back what I think, I want to hear what they think.

Figure 5-1 represents the sentences that I read to them and my reflections on what students said about agency in each sentence. There appears to be some repetition in the third, fourth and fifth sentences. Two of the singer's utterances (sentences 3 and 4 in this table) are embedded in a sentence written by the article's author (sentence 5). Each of these three sentences was considered independently.

Even in this second week of our interaction, Joey was already the most vocal of all the students. He did not mind disagreeing with me. Indeed, he seemed to take pleasure in it. I thought he could be the very person Walkerdine (1988) had in mind when she wrote:

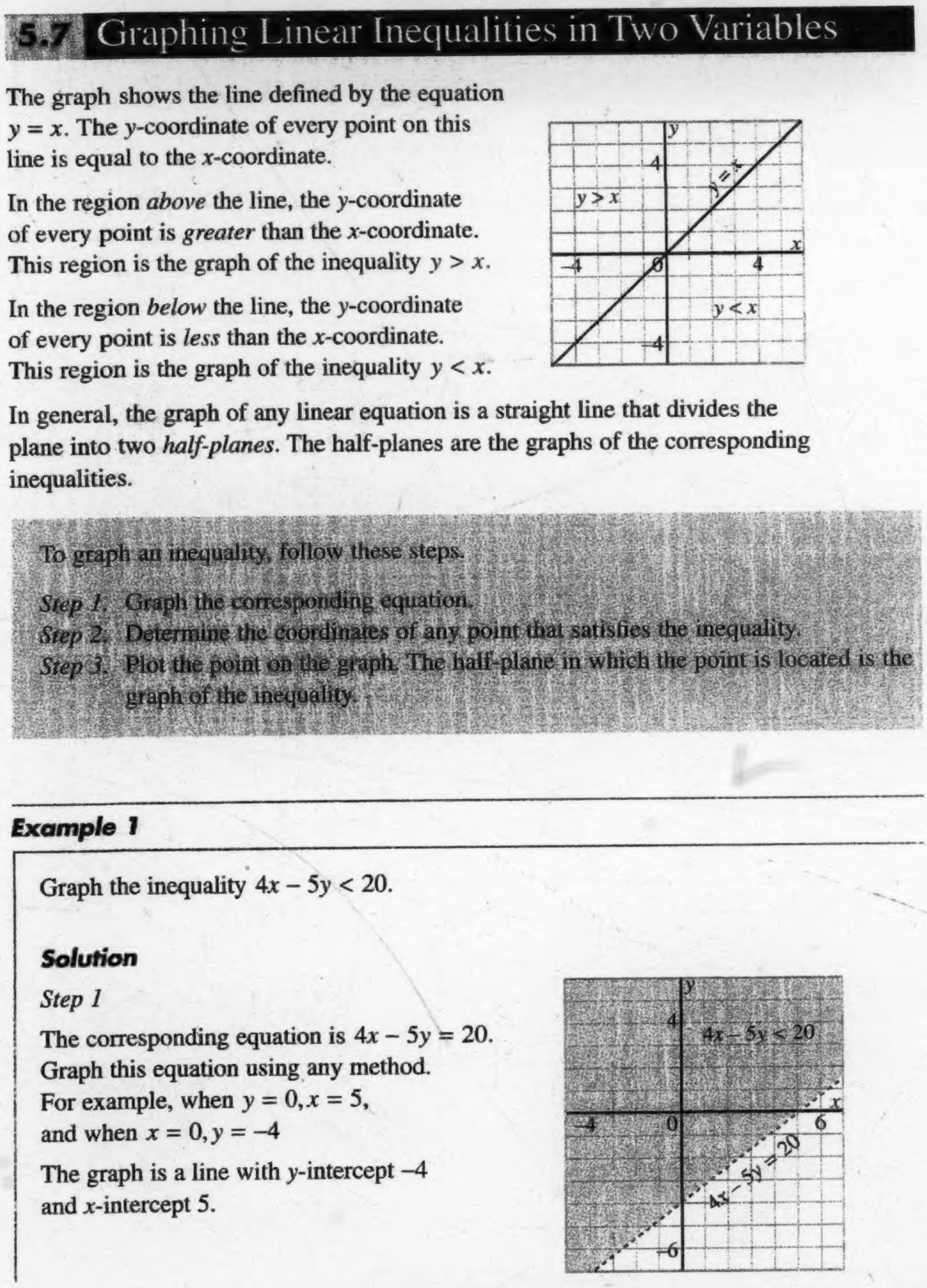
> While conflict and rule-challenging provide proof of masculinity, only that directed at the overthrow of the discourse itself, the teacher's right to 'mastery', provides the […] evidence of real understanding or 'brains', 'brilliance', and so forth. [...] The challenging is difficult for it threatens the teacher's control and yet simultaneously provides the desired evidence. (p. 209)

It did not surprise me that Joey eventually failed this mathematics course. Despite his displays of "brilliance" in class, he tended not to succeed on formal assessments, which tested his rote knowledge and rule-following. At times, Mrs. Hill found Joey's propensity to challenge his teachers exasperating.

	Sentence from Newspaper Article	Student Response
1	Just two days after cleaning up at the Canadian Country Music Awards, Shania Twain was already back at work preparing for her world tour, which kicks off Sept. 25 at Hamilton's Copps Coliseum.	Shania Twain has agency.
2	Twain will rehearse at the Steeltown venue for the next two weeks.	Twain has agency.
3	"The location is good."	Joey said that "the location" has agency. I told him that inanimate objects cannot have agency, that some sentences just have no agency. He said he disagreed, but he did not pursue the conflict.
4	"Several of our first dates are in Canada."	No agency.
5	"*The location is good. Several of our first dates are in Canada [as shown above]*" said Twain, dressed casually in sweats and runners.	Twain has agency.
6	"A lot of it has to do with availability too, and the time frame."	No agency.
7	Twain plays Ottawa on Sept. 27 and Toronto on Oct. 2 and 3.	Twain has agency.
8	The last time the Timmins, Ont.-raised singer hit the road, in 1998, she launched her tour in Sudbury.	Twain has agency.
9	So might there also be a patriotic bent to the launch city too this time?	Jessye noted that this is a question and wondered if they have agency. I said they could have implied agency.

Figure 5-1. Introducing agency using popular press text

On the school day following our analysis of the newspaper article, I asked the students to watch for agency in our mathematics class. At the time it seemed that they thought agency was not important, because when they looked at their textbook they could not find examples of humans with agency. With four minutes left in the first of two periods, I asked the students to look for agency on a particular textbook page that related to the topic we had been discussing: "Who makes things happen?" I asked. Figure 5-2 is a copy of this page.



5.7 Graphing Linear Inequalities in Two Variables

The graph shows the line defined by the equation $y = x$. The y-coordinate of every point on this line is equal to the x-coordinate.

In the region *above* the line, the y-coordinate of every point is *greater* than the x-coordinate. This region is the graph of the inequality $y > x$.

In the region *below* the line, the y-coordinate of every point is *less* than the x-coordinate. This region is the graph of the inequality $y < x$.

In general, the graph of any linear equation is a straight line that divides the plane into two *half-planes*. The half-planes are the graphs of the corresponding inequalities.

To graph an inequality, follow these steps.

Step 1. Graph the corresponding equation.
Step 2. Determine the coordinates of any point that satisfies the inequality.
Step 3. Plot the point on the graph. The half-plane in which the point is located is the graph of the inequality.

Example 1

Graph the inequality $4x - 5y < 20$.

Solution

Step 1

The corresponding equation is $4x - 5y = 20$.
Graph this equation using any method.
For example, when $y = 0, x = 5$,
and when $x = 0, y = -4$

The graph is a line with y-intercept -4 and x-intercept 5.

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Figure 5-2. Page 348 of *Addison-Wesley Mathematics 11* (Alexander and Kelly, 1998)

In this account, I draw upon my hand-written account of our conversation, which I recorded immediately afterwards. After I prompted them to look for agency, Joey responded, "the graph" to the implied question – "Who has agency?" Gary jokingly added, "Addison-Wesley" (the textbook publisher). Otherwise it was basically quiet for a while.

I asked if there were any people doing anything. Students shook their heads – no. I asked why not? Signot said it was "because it's numbers" and then added "or graphs." I wondered later whether he said *it* because it was about numbers or because it was numbers acting as subjects. At the time, I thought he meant the graphs and numbers were acting as subjects.

I then asked the class, "So, what are we doing then? The numbers make us do things?" There were no answers to this question. Joey diverted attention from the language in the textbook to himself. He seemed to be making a joke of the lack of personal agency by saying, "We can use people instead of numbers." He read from the book, replacing "*x*"" with "Kari" and "*y*" with "Gary."

This was the end of our discussion about agency in the textbook. The students did not seem very interested in the issue. In my reflections that afternoon, I described my sense of their response to the agency-related tasks I had given them.

> When I talked about agency today and Friday I felt like I am [*sic*] talking into the wind. Joey answers, but I wonder if he sees it as his chance to perform. Signot also responds. Gary says a bit. Matt looks attentive but doesn't say anything.

Everyone else seemed to be inattentive. They chatted amongst themselves, showing their lack of interest. After being silenced by most of the class for two consecutive discussions about agency, I decided to give in for a while. Though I considered the absence of personal agency in their textbook significant, I did not immediately resist the students' apparent lack of interest.

As I considered the first sentence on this textbook page – "The graph shows the line defined by the equation $y = x$." – I wondered and continue to wonder how a graph can show something to a person and how an equation, independent of people, can define something. Is it not a person using a graph to show someone something and a person using an equation to define the line? Personal agency is masked in these excerpts from the textbook. Surely this should be interesting to these students, I thought.

The masking of agency seems to be an obvious issue in the study of language form. The use of the passive voice to hide agency is called by Jensen (1989) a "linguistic danger signal" in the context of interviews. These are strong words – "danger signal." Stubbs (1996), in demonstrating the critical value of his methodological imperatives, also suggests the importance of agency by choosing to analyze a linguistic form that hides agency. He draws attention to *ergative verbs* – verbs for which a noun can be placed as either the subject or the object of a sentence – and shows how they can be used to mask personal agency. Because the aim of his paper is to exemplify the power of his particular methodological approaches, it seems that his decision to use a language form that masks agency speaks to its obvious importance.

Hidden agency is also important to mathematics educators. In her analysis of lexico-grammatical features of scientific discourses, Morgan (1998) notes the frequent use of the passive voice, but reports that there are also personal, expressive forms in these discourses:

> In spite of the common perception that the impersonal is the rule, however, there is some evidence to suggest that it is by no means universal and that there are, in fact, systematic, purposeful ways in which personal forms are used within formal academic writing. (p. 15)

It seems that impersonal forms predominate in mathematics textbooks. In an analysis of the Alexander and Kelly (1998) textbook, which we were using in my researched classroom, and a competitor's textbook, I noted that the passive voice dominates in both textbooks and I expressed my concern that the extensive use of the passive voice devalues the personality and experiences of students and implicitly describes mathematics as a set of pre-existent facts and predetermined procedures (Wagner, 2002c).

In his critique of Fairclough's critical discourse analyses, Widdowson (2000) questions Fairclough's (1989) evaluation of hidden agency. Though I was conscious of Widdowson's reminder that "referential *avoidance* is not the same as referential *evasion*" (p. 166, emphasis his), it seemed clear to me during my discussion with my participant students that there was an important issue just waiting to emerge in our conversation about agency. At this early stage of the larger conversation about language practice in their classroom, I was content to let the issue emerge naturally. The students' lack of interest in agency in mathematics discourse contrasted with my passionate interest; I felt tension in this part of our conversation, tension that I suggest was generative.

In this initial interaction about agency, I did not give the students a definition of the word *agency*. Instead, I wanted their sense of the word to develop from their own use of it. Nevertheless, my simple question, "who is said to be making things happen" was similar to Pickering's (1995) description of agency. He describes choice and discretion as the classic attributes of human agency, and passivity as its antithesis. I did not discuss with the students *how* they discerned the agency in each sentence. In the conversations that followed, we focused our attention on the voice of sentences. For example, if the subject of a sentence is the first person, *I*, then the speaker is likely to be taking initiative in some way.

Introducing Student Texts

A few weeks after our initial discussion about agency, Mrs. Hill gave students a written test that included the task shown in Figure 5-3.

> Consider the quadratic function $f(x) = (x-1)^2 + 3$. Explain how you can tell which of the following is its inverse:
>
> $y=\sqrt{x-1}+3$ $\quad y=\sqrt{x-3}+1$ $\quad y=\pm\sqrt{x-1}+3$ $\quad y=\pm\sqrt{x-3}+1$

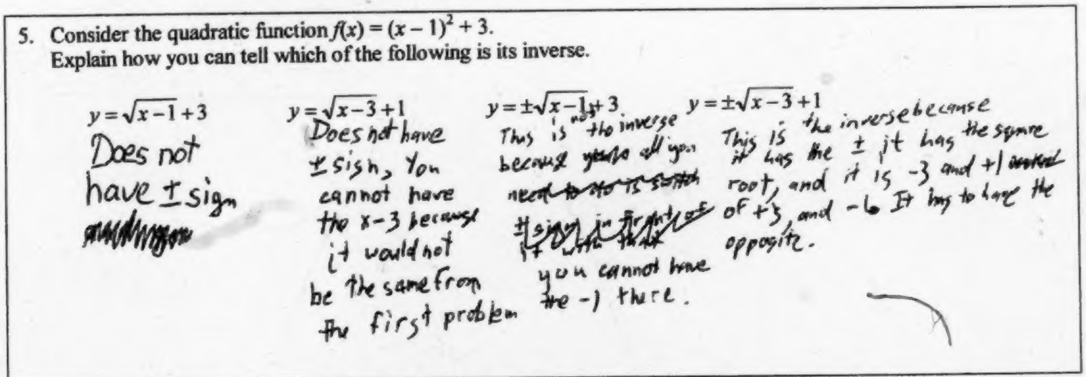
Figure 5-3. A test question that asks for explanation

The next day, Mrs. Hill returned the graded papers and I went through the test with the students, writing on an acetate sheet on the overhead projector. When it came to the question in Figure 5-3, the only question to ask students for explanation, I used a prepared overhead slide to show the students some samples of their responses. Without indicating that I wanted to talk about language features in their writing, I showed and described two longer responses (Figure 5-4).

I had retyped the text from the students' actual responses and printed them with a generic font in order to conceal the identity of the writers. This interpretive act would have an effect on the way students saw these texts. With the clean and even font, it was easier to attend to word choice and more difficult to attend to other features of the students' writing. Compare the first of these two responses with the image of Terry's actual writing, shown in Figure 5-5.

under equation 1 –	"Does not have ± sign"
under equation 2 –	"Does not have ± sign. You cannot have the x – 3 because it would not be the same from the first problem"
under equation 3 –	"This is not the inverse because you cannot have the –1 there."
under equation 4 –	"This is the inverse because it has the ± square root and it is –3 and +1 instead of +3 and –1. It has to have the opposite."
[*has a table of values for each of the five equations including the original function, and concludes at the bottom of each table "not opposite" for the first and third potential inverse functions and "opposite" for the second and fourth potential inverse functions.*] It must have opposite coordinates with the $\pm\sqrt{\ }$.	

Figure 5-4. Representations of Terry's and Kyle's responses


5. Consider the quadratic function $f(x) = (x-1)^2 + 3$.
Explain how you can tell which of the following is its inverse.

$y = \sqrt{x-1} + 3$
Does not have ± sign

$y = \sqrt{x-3} + 1$
Does not have ± sign, You cannot have the x-3 because it would not be the same from the first problem

$y = \pm\sqrt{x-1} + 3$
This is not the inverse because ~~you're all you need to do is switch ± sign in front of it with that~~ you cannot have the -1 there.

$y = \pm\sqrt{x-3} + 1$
This is the inverse because it has the ± it has the square root, and it is -3 and +1 ~~instead~~ of +3, and -1. It has to have the opposite.

Figure 5-5. Terry's response

Initially, I showed only the first response and talked with students about how the student authors of these samples answered the given prompt. Then I revealed the second sample and led a similar discussion.

There are a number of interesting lexico-grammatical choices revealed in these student texts, including the deictic vagueness of the words *it* and *there*, the high modality in Kyle's response – "It *must* have …" – and the expressions of agency indicated by personal pronouns. I wanted to focus on one particular issue – agency. After talking about the two texts separately, I displayed a set of four transcripts of student responses to the same question (Figure 5-6).

We switch around the x & y (inverse) & do the work. [*calculations accompany the explanation*]
Switch the y and the x and find the value of y. [*calculations accompany the explanation*]
You switch the x and y and then solve for y which will give you the equation of the inverse line. [*calculations accompany the explanation*]
I can tell by switching the y and the x in the original equation & then solving for y. [*calculations accompany this answer*]

Figure 5-6. From Charlene's, Kalli's, Joey's and Signot's responses

The following transcript represents my interaction with the students after I revealed these responses.[8]

DW: Do you notice anything interesting from these four?
Kyle: They all use the word *switch.*
DW: They all use the word *switch.* What word is different in all of them? [*long pause*] Do you notice this? [*circling the personal pronouns in the four responses* – we, you *and* I]
[*Mrs. Hill and a few students laugh emphatically*]
DW: So, isn't that kind of interesting? Which is, which is best?
Rory: The second one. [*simultaneous*]
Signot: The one with *I.*
DW: You think *I* is best? [*looking at Signot*] That's what you wrote too, isn't it?
[*a few students laugh*]
Rory: I think the second one is best
DW: Mrs. Hill, which do you think is best?
Mrs. Hill: Um, I'm not sure. [*laughing*]

DW: And then there's the one who avoids, who does just the switch, avoids the people altogether, right?
Rory: Yeah. [*laughing*]
Signot: It's *you* who did it | so
DW: | What do you | think's best, Joel?
Matt: Well, obviously it asks you to tell how *you* can tell which of the following is its inverse. So you're not saying, "Well my partner, the guy sitting beside me, is."
DW: Yeah. So you wouldn't say, um, well, "Joel."
Matt: | "Joel, who I copied off of." | [*simultaneous*]
DW: | That would be copying, yeah. |

Transcript 5-1. Noticing varying voices in student writing

Reflecting on this interaction, I regret not having noticed Rory's contributions. She was quite persistent, answering three times and then talking with me after class. She was close to the recording device, so while I did not notice it at all during the dialogue, her voice was clear on the video record.

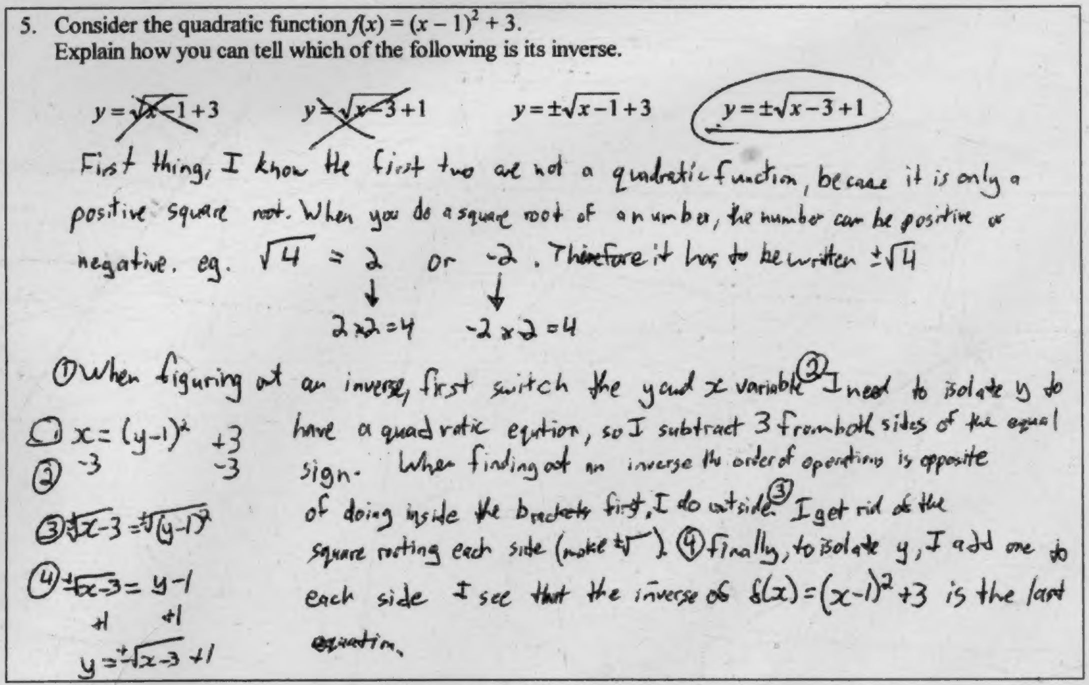
I would have been interested in Rory's reasons for promoting the second student response. After class she told me that she preferred it because it was hers. In reality, it was not hers. It was her friend Kalli's. Rory had written, "Is it's [*sic*] inverse because the (p) and (q) are reversed." Her apparent sense of ownership of her friend's response is interesting. Perhaps she had reasons for preferring Kalli's response, reasons beyond loyalty. Could it be that she was not claiming ownership of the actual response, but rather claiming ownership of the voice of the response? Both her response and Kalli's were devoid of explicit reference to people, though Kalli's imperative form tacitly implies an audience in this particular context. We might command a pet dog and say "sit" or we might try to command inanimate objects, as King Canute tried to command the sea to go back or as a computer programmer encodes "Goto [line] 350," but Kalli's imperative has her subject doing mathematics, which presumably can only be done by a person. I am particularly interested in Rory's reasons for persisting in her preference for texts with no explicit reference to people, because this preference would emerge as a class preference in days to come.

Unlike Rory, Signot gave a reason for preferring his own written response; he suggested that the *I* voice is most appropriate because he was reporting on his personal approach to the task – "It's *you* who did it." He seems to have noticed his own agency and considered it appropriate to reveal that agency in his subsequent writing. However, his preference for the *I* voice seems to have been short-lived. When his classmates began championing mathematical utterances devoid of reference to people, Signot did not speak in favour of the *I* voice. Though it was common in this class for students to say nothing, Signot usually spoke freely. He participated frequently in discussions about language, and especially in discussions about mathematics.

Another interaction I had with Signot might also suggest his general preference for non-referential utterances. At the end of the term, I talked with some students about pseudonyms. I explained to this group that in my research reporting I typically try to choose pseudonyms that are similar to participants' actual names, sharing ethnic origin. Signot asked me if he could be called *Signot*, which he said was a proper Russian name that also means *anonymous*.[9] Is there a connection between choosing to be represented by a name that means *anonymous* and supporting mathematical utterances that mask the people behind them?

Matt, the other student who expressed his preferred response, noticed the connection between the prompt and the responses. He seems to have spoken in support of the *I* voice. Apparently, he noticed the voice-leading nature of the question's wording – "Explain how *you* can tell …" – and suggested that it is natural to answer a *you*-question with an *I*-answer. Unlike Signot and Rory, he gave a reason for his preference, but did not explicitly state what his preference was.

Like Rory, Matt supported the response that was most similar in voice to his own response. His response began with an *I* voice – "First thing, *I* know the first two are not a quadratic function..." – but he did not continue with this voice consistently. He continued with "When *you* do a square root." Figure 5-7 shows his response.



5. Consider the quadratic function $f(x) = (x-1)^2 + 3$.
Explain how you can tell which of the following is its inverse.

$y = \cancel{\sqrt{x-1}} + 3$ $\quad$ $y \cancel{= \sqrt{x-3}} + 1$ $\quad$ $y = \pm\sqrt{x-1} + 3$ $\quad$ $\boxed{y = \pm\sqrt{x-3} + 1}$

First thing, I know the first two are not a quadratic function, becaue it is only a positive square root. When you do a square root of a number, the number can be positive or negative. eg. $\sqrt{4} = 2$ or -2. Therefore it has to be written $\pm\sqrt{4}$

$2 \times 2 = 4$ $\quad$ $-2 \times 2 = 4$

① When figuring out an inverse, first switch the y and x variable ② I need to isolate y to have a quadratic eqution, so I subtract 3 from both sides of the equal sign. When finding out an inverse the order of operations is opposite of doing inside the brackets first, I do outside. ③ I get rid of the square roting each side (make $\pm\sqrt{\ }$). ④ Finally, to isolate y, I add one to each side I see that the inverse of $f(x) = (x-1)^2 + 3$ is the last equation.

① $x = (y-1)^2 + 3$
② $-3 \qquad -3$
③ $\pm\sqrt{x-3} = \pm\sqrt{(y-1)^2}$
④ $\pm\sqrt{x-3} = y-1$
$+1 \qquad +1$
$y = \pm\sqrt{x-3} + 1$

Figure 5-7. Matt's response

After hearing these various opinions, I offered an interpretation that I hoped would provoke resistance, but I gave no time for response. I said that the subjects of these sentences were interchangeable. In mathematics, it does not matter who does something, because the result should be the same no matter what. I summed up by saying, "In mathematics, people don't matter."

My plans to pursue this conversation about voice and to relate it to issues of agency were foiled by various disruptions. A four-day long weekend and an extra-curricular class engagement intervened. Also, on the first day after our discussion about the various voices, I had planned to prompt a discussion about agency by interrupting the flow of a mathematics conversation when students used pronouns in interesting ways. I failed to do this because I had trouble noticing language use in action and because I did not want to interrupt important mathematics (a reason I discussed extensively in Chapter 4).

Upon reflection, I realized that I myself was experiencing difficulty doing the very thing I wanted the students to do – pay attention to their language practice while using that language for mathematics. Because I was concentrating on communication *about* mathematics, language itself was for me at this time a transparent, non-problematic medium.

Adler (2001) extends Lave and Wenger's (1991) idea that resources need to be "transparent" for people fluent in a practice. Though Lave and Wenger do not include language as a resource, Adler suggests that students should be able to use language transparently. Students in multilingual classrooms, the context in which her work is situated, need to gain access to language as a resource. However, they also need to develop facility with using language transparently. Adler illustrates the dilemma facing a teacher in this kind of setting: when to direct attention to language itself and when to use language as though it is transparent. She calls this tension the "dilemma of transparency."

> The horns of this dilemma are that, on the one side, explicit mathematics language teaching, in which teachers attend to learners' verbal expressions as a public resource for class teaching, appears to be a primary condition for access to mathematics, particularly for learners whose main language is not the language of instruction. On the other side, however, there is always the possibility in explicit language teaching of focusing too much on what is said and how it is said. (Adler, 2001, p. 115)

The dilemma is also present in unilingual classes because students need to learn new vocabulary and mathematical genres. There is always the option to direct attention to language, but it is not a dilemma unless it is a difficult decision. To be a dilemma, each choice has to have both value and problems.

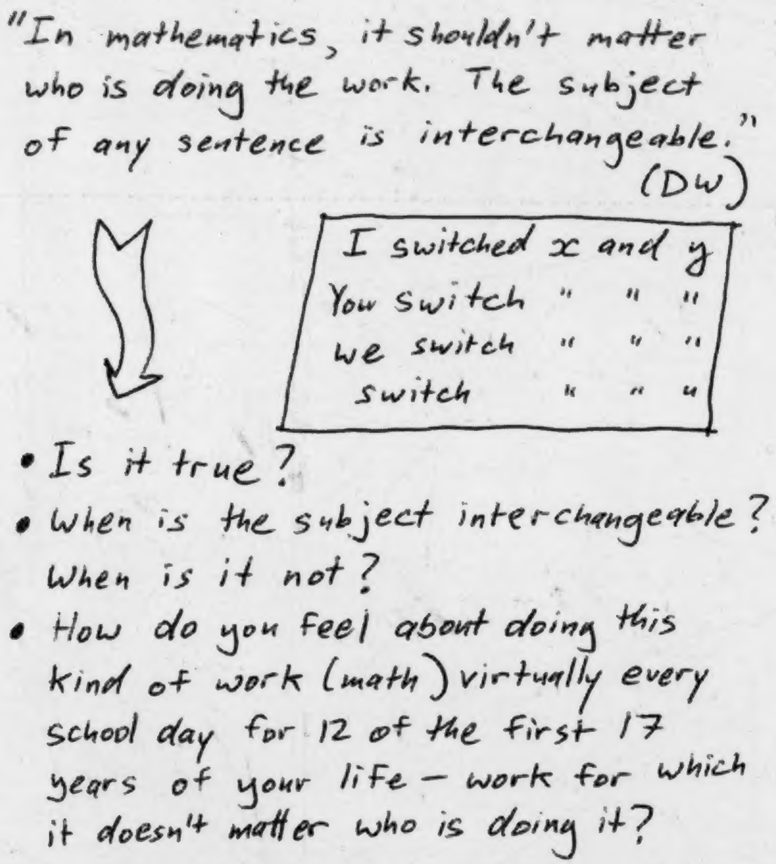
Debates about teaching the features of genres explicitly, such as the one between Burton (1996) and Solomon and O'Neill (1998) I described in Chapter 2, testify that there is a dilemma outside multilingual mathematics classrooms. Solomon and O'Neill promote the explicit teaching of certain forms of mathematical writing, and they gloss Burton as promoting a narrative form. Morgan's (1998) investigation of the writing-to-learn movement also relates to the dilemma between attending to language and attending to mathematics. She wonders if teachers should tell students which language forms are likely to be favoured by their external assessors. Indeed, her treatment of the dilemma leads her to consider the possibility of using critical language awareness in mathematics classrooms.

While I was trying to direct the participant students' attention to language, I myself was having difficulty doing this. I believed that there would be value for students in becoming more aware of language as a mediating resource, but I realized that there was little space in the discourse patterns to direct attention to language. I had to create the space.

To avoid failing once again to notice the students' personal pronoun use in class, I began the next day by continuing the conversation about pronoun use. The students immediately engaged in the conversation even though it had been a week since our brief discussion of voice in their writing. I put the same group of four student responses (Figure 5-6) on the overhead projector, with the initial pronouns still circled. After they said they recognized this set of utterances, I put another slide on the projector (Figure 5-8), and said:

> We had the "we switch," "switch," "you switch" and "I can tell by switching," and I said something to you about that. I said in mathematics, it shouldn't matter who is doing the work. The subject of any sentence is interchangeable. Remember me saying that?

Most students nodded yes. Next, I read aloud the questions from the overhead projection, laughed and added after the last question, "I know that's a loaded question. Any answers to any of the questions?"



"In mathematics, it shouldn't matter who is doing the work. The subject of any sentence is interchangeable."

(DW)

I switched x and y
You switch " " "
We switch " " "
switch " " "

- Is it true?
- When is the subject interchangeable? When is it not?
- How do you feel about doing this kind of work (math) virtually every school day for 12 of the first 17 years of your life — work for which it doesn't matter who is doing it?

Figure 5-8. Overhead slide with questions about agency

As I was asking for students to answer any of these questions, I noticed Matt smile with understanding. He is an athlete who was playing on a number of school teams. He recognized an opponent trying to provoke him into a fight. I began the discussion by directing attention to him, and saying "I see a smile from one." Matt smiled even more but said nothing. Then Joey put up his hand, indicating he wanted to speak:

DW: Okay, Joey?

Joey: One. It is true.

DW: It is true. Okay, do you want to say more, or is that, that's all?

Joey: Oh. I'm just saying whether Joey does the work or Gary does the work. It doesn’t matter. That’s why Joey doesn’t work.

DW: [*laughing*] Any other answer to one of those questions?

Signot: It doesn’t matter.

DW: Signot?

Signot: I think we shouldn’t have to explain if the answer is right.

DW: If you have an answer, it’s, it’s what?

Signot: It’s probably good.

DW: It’s good enough?

Signot: Well, if you have a good answer the work should be good.

DW: You should be able to explain it, yeah.

Signot: No, but good work explains itself.

DW: Oh, the work explains itself?

Signot: Yeah.

DW: So you shouldn’t have to use words to explain it. Ah. I bet some people would disagree with that. I can see one right there. [*laughing and pointing at Mrs. Hill, who nods agreement*] Okay, one more comment and then I’ll go away.

Joey: Okay, one more thing. I think you should, well personally, I think you shouldn’t use *I*, *you*, or *we* or *me* or whatever because if you say, “you switch,” that means that somebody else has to do something different. You know what I’m talking about?

DW: To, telling someone what to do.

Joey: No, because if you like, “you switch” something and if somebody else decides to not switch you’re making that one person switch it. It’s all wrong. Shambles.

DW: So, you think we should just get rid of the *I*’s and the *you*’s and the *we*’s.

[*Gary shakes his head no*.]

DW: Gary, Gary disagrees.

Gary: No, I agree with Joey.

DW: You do? Okay. [*packing up my stuff and walking to the back of the room*]

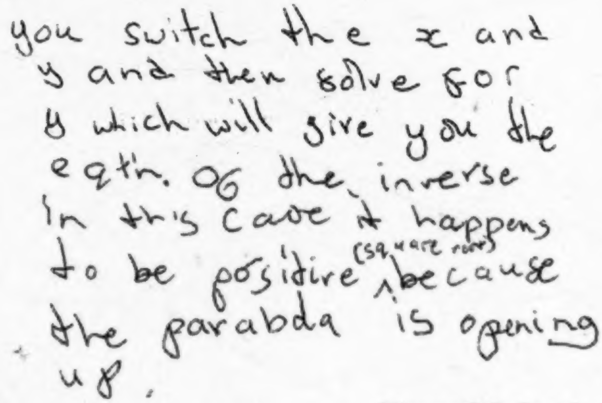
Mrs. Hill: [*approaching the front of the class*] I don’t really care what you do. Just get the answer right.

[*Mrs. Hill and some students laugh.*]

Transcript 5-2. Students express their opinions about voice

Joey and others articulated a literal interpretation of these personal pronouns. They did not seem to see the possibility of *you* being used in a general sense. Rowland (2000) has noticed "the use of the vague, unmarked 'you', functioning as a vague 'generalizer'" (p. 109), but he claims that "this matter seems to have escaped analytical attention with regard to English speech" (p. 76). No wonder Joey and his classmates did not have a ready way of describing their use of the pronoun. Bills (2002) gives evidence that higher-achieving students use *you* in this general way, but there is no indication that the students in this research consciously used the generalizing form of *you*.

It became clear to me that the students in my research group did not see this usage as a possibility. I saw an opportunity to help these students become aware of a practice to which they had already been exposed. Indeed, they themselves regularly used *you* in a general sense without appearing to realize it. They regularly mixed *you* with other personal pronouns. Notice, for example, Matt's response to the test question (Figure 5-8), in which he switched back and forth between voices, beginning with an *I* voice ("I know the first two..."), switching to a general *you* voice ("When you do a square root..."), then to imperative ("first switch the *y* and *x* ...") and back to an *I* voice ("I need to ..."). For a more poignant example, notice that although Joey spoke against the *you* voice, he employed it in his response to the mathematics question that prompted the student writing under consideration (Figure 5-9): "You switch the *x* and *y* ...."



you switch the x and
y and then solve for
y which will give you the
eqt'n. of the inverse
In this case it happens
to be positive (square root) because
the parabola is opening
up.

You switch the x and
y and then solve for
y which will give you the eqt'n. of
the inverse. in this case it happens to
be positive (square root) because
the parabola is opening up.

***Figure* 5-9**. Joey's response

In the coming days, I would point the students' attention to more of their own practice.

Introducing a Reflexive Set of Student Texts

In the next class period, I challenged Joey by quoting excerpts from his utterances about voice and from his subsequent participation in mathematics discussion. On this day, I introduced a rule that no student could participate two consecutive days in discussion about language. My aim was to promote wider participation. I explained the rule before quoting Joey.

DW: On Thursday, I asked you about agency in math, the "I switch," "you switch," "we switch" thing [*putting the overhead transparency on, with the following quotations written on it*] – and Joey said, "I think you shouldn't use *I* or *you* or *me* or *we* or whatever because if you say 'you switch' that means somebody else has to do something different." Then, ten minutes later, he said, "say you are on a test, what would you round this one to?" And he did the exact opposite of what he said.

[*some students laugh uncomfortably*]

DW: So, now it looks like

Joey: I [*putting up his hand*]

DW: Oh, you can't talk. [*indicating "stop" with a hand gesture*]

[lots of laughter as Joey smilingly makes a threatening gesture with his fist]

DW: Yes, I realize, but we're going to give you the last word.

Joey: Okay. [*suddenly becoming very calm*]

DW: Okay. But I want to hear other people first.

Joey: Okay.

DW: But, actually I'll tell you that I see a lot of significance in what Joey said, a lot of truth. But the question I want to ask you is: What's going on with the contradiction? [*long pause*] Yup? [*gesturing that Matt should speak*]

Transcript 5-3a. Analyzing Joey's contradiction

As I had predicted in a reanalysis of interview data from my master's research (Wagner, 2003b), students felt uncomfortable when their words were quoted back to them. Some of Joey's classmates laughed their sympathetic embarrassment when I pointed out the contradiction. I was acting like a crown prosecutor cross-examining the accused. Joey's charismatic reaction to the contradiction and "our" new rule defused the situation nicely. By the time I finished speaking my reminder, Joey was half standing with a huge smile and shaking a mock threatening fist at me. He grunted and groaned, giving the sense that he wanted to speak and that he was exercising supreme self-control to hold back his comments. After the laughter died down and Joey sat down quietly, showing that he had resigned himself to not speaking, a classmate defended Joey's word choice by describing how Joey was addressing a particular person. Matt tried to explain Joey's intention in his mathematical utterance. Of course, Matt had no access to Joey's intentions, only to his utterance. No one has access to another person's meaning, only to the form and content of the other's expression. (As the conversation continued, a student whose voice was not recognizable in the audio and video recordings joined in.)

Matt: Well, Joey, Joey's just asking you what you think with his question.

DW: Okay. All right. Let me just get my notebook from yesterday. In which I wrote down, yesterday Joey said, um, "you know you have degree two blah blah blah blah." That's a different situation. But you're right about this one, obviously. [*pause*]

girl: Joey always speaks in *you* or in third person, so it doesn't really matter.

Joey: You know what Joey does, is like.

[*many students laugh*]

DW: He does refer to himself that way. It's true. Does anyone else do that though, or is it just a Joey thing?

Signot: Joey thinks that Joey

Joey: Joey does.

DW: It is normal. We all do it. Do you ever, do you notice people on the news using *you* like that? When they're telling you about a situation and they say um, "first I" um you know, maybe they're interviewing someone who went through a bombing attack. And they'll say, "first I um jumped for cover and then you know, when you're in a situation like this and you've got blah, blah, blah." Do you ever hear a situation like that, where they switch from *I* to *you*? When do they do that.

Signot: When they try to explain to relate to you.

DW: To who? The interviewer?

Signot: No.

Tharshini: Yeah, when they want the other person to really understand.

DW: Ah. Any other situations? [*pause*] Okay in math it happens more than anywhere actually. And some people say that when I say "you do this" and "you do that," it means "anyone." This just refers to anyone. When I say *I* it refers to me. So, yeah Matt?

Matt: Well it possibly makes sense when you're explaining how to do something. So, um, if you use *you* and stuff it's telling them what, since they're doing it, like.

DW: Yeah.

Matt: So if it's putting pretty much what you did from their perspective.

DW: It's easier to see from, for the person you're talking to.

Matt: I think so.

DW: So, Joey, do you want the last word?

Joey: Oui. Okay, could you just read the last sentence I said?

[*DW looks at the overhead projector to read.*]

Joey: No, like before.

DW: Oh. [*picking up the notebook to read*] "You know you have a degree two because."

Joey: I was simply stating what *you* know.

DW: Oh. But

Joey: I assume that you're a genius.

DW: Okay. All right. Let's go on. [*shifting attention to the day's mathematics content*]

Transcript 5-3b. Analyzing Joey's contradiction

Matt explained how Joey's use of the word *you* was literal, suggesting that he was talking to me rather than speaking generally – "Joey's just asking *you* what *you* think" (emphasis added). (Actually, Joey had been addressing Mrs. Hill, not me, but the students often saw us as interchangeable.) I responded by quoting another more obvious instance of Joey using *you* in what I considered to be a general sense – "you know you have degree two." Signot and Tharshini also seemed to support Matt's interpretation. I sensed that they were telling me that this *you* voice represented an individual trying to relate his own experiences in such a way as to help others understand his experiences from their own perspectives. I resisted their interpretation and gave an invented example of people using the general *you* voice in everyday life, but I did not call it a "general" voice. And the students likewise resisted my interpretations, giving plausible literal interpretations of the pronoun *you* for both my mathematics classroom examples and my everyday example.

On reflection, I can see how their interpretations actually did describe the general sense of the pronoun *you*. These students were describing mathematical communication as an attempt to combat diversity of perspectives. They were describing a discourse that promotes a sense of everyone seeing the same things in the same way. This explains the general *you* voice and even the mathematics class *we* voice that Rowland (2000) and Pimm (1987) remark on and discuss.

Mühlhäusler and Harré (1990) suggest that in academic discourses authors employ a *we* voice when they are trying to draw their readers into complicity, hoping to trap them in tacit agreement. Rowland (2000) believes that mathematics teachers employ the *we* voice with similar intent. Pimm (1987) also refers to the normalizing force of the inclusive pronoun *we*: “The least that is required is my passive acquiescence [...] I am persuaded to agree to the author’s attempts to absorb me into the action” (p. 73). What about mathematics students? Would they have such a normalizing agenda? I prefer Pimm’s other explanation for the use of the *we* voice. The *we* can refer to the anonymous collective of mathematicians, people who do things right. It refers to convention.

Students who want to demonstrate their membership in this collective of people who do things right have the *we* voice at their disposal: “We switch the *x* and *y* variables to find the inverse.” Yes, a real collective, a real *we* does find inverses in this way. *We* is literal. *You* is too: “You switch the *x* and *y* variables to find the inverse.” If *you* want to be part of the collective, the people who do things right, *you* will do things the way the collective does things. *You* is literal when it is used in its generalizing sense.

The literal usages of these pronouns refer to convention, to what Bakhtin (1953/1986) calls the neutral word. Though it can be read as literal, Bakhtin characterizes the neutral word as belonging to no one. These "personal" pronouns can be taken as literal and as general affectations at the same time. My participant students were not wrong to find literal explanations for the pronouns. Though I wanted them to see that these words refer to an abstracted other, I realized in reflection that their interpretation and mine were not incompatible.

My interpretation of the students' emerging understanding was supported a week later in an interview with Joey, when he made the proclamation I quoted at the beginning of this chapter: "You shouldn't use any voice, ..." After I reminded him about my public display of his contradiction, he answered:

> Well, about that, I said, I said you shouldn't use any voice, you should use the general voice. I've termed it the general voice because I'm cool and I can make my own terms. Like this is to be done and that's it. When, if I said that I'm going to do it, that means I'm going to do it. That means somebody else has to do it differently. That's why I don't really like using the *I* terms, the *we*'s and the, I just prefer using: this is what's supposed to be done, and then just go ahead and do it.

He seems to have been sympathizing with the sense of compulsion felt by Mühlhäusler and Harré (1990), Rowland (2000) and Pimm (1987), when he was speaking against the use of personal pronouns in a generalizing way: "That means somebody else has to do it differently." Joey preferred an absence of human reference in mathematical utterances.

Joey's proclamation is significant when it is considered in its context. With this introduction of his own terminology, he demonstrated his individual human agency, his capacity to explore varying ways of participating in mathematics discourse. Later in the interview he expressed surprise when I told him that scholars have also used the word *general* to describe utterances that refer to no one in particular. And he reiterated his sense of personal agency: “Well, I’ve never heard it before. So, to me, I came up with it. [...] It does make sense, because this is generally speaking.”

In addition to displaying his human agency in the production of language, he touched upon important characteristics of mathematical thinking – generalization and abstraction. His move from envisaging particular perspectives in mathematics to envisaging a general, conventional perspective exemplifies a tension that is at the heart of mathematics. Nevertheless, I still wanted him to see a place for personal, expressive voices in mathematics, especially because he was the most expressive of the students in his class.

The Expressive Form of the Dance of Agency

Pickering (1995) identifies the tensions associated with agency in his account of historical scientific and mathematical advances. He identifies different types of agency – human, material and disciplinary – but he does not consider material agency to be significant in mathematics.[10] Human agency can be resisted by physical reality (material agency) or by conceptual systems (disciplinary agency). When scientists and mathematicians follow the established conceptual patterns of their disciplines they surrender to disciplinary agency. It is when they take initiative with open-ended modelling and cross-discipline conversation that they extend present cultural and conceptual practices and in so doing demonstrate their human agency. He calls the tension between human and disciplinary agency in such instances a “dance of agency.”

Boaler (2002, 2003) draws on Pickering's metaphor to describe good mathematics class discourse. While Pickering is interested in global cultural extension, Boaler is more interested in local extensions of discourse, particularly in mathematics classrooms. In her depiction of traditional classrooms, students simply follow the paths set before them. They surrender to the classroom disciplinary agency. By contrast, she promotes classroom discourse that prompts students to take initiative, to demonstrate human agency. Unlike Pickering's interest in scientific, disciplinary advancement, it seems that Boaler's interest lies in each individual mathematics student's human advancement. I suggest that the strong presence of the student *I* voice in Boaler's (2003) exemplar demonstrates human agency within the classroom disciplinary setting. In Boaler's exemplary class, the student Ryan moves from an *I* voice to a generalizing *you* voice as he describes his move toward generalization:

> *I*'m trying to find a general formula for the, this triangle (obtuse). Because *I* knew that the triangle used to find the height is right there [...] *I* did um 180 minus theta, because if *you* know, if *you* know that angle right there is theta, then *you* know that the two combined have to be 180. (p. 10, emphasis mine)

Concurrent with this student's move from the *I* voice to *you* voice, there is a shift from past to present tense. The utterance suggests that the student first saw himself doing mathematics, using the past tense and *I* voice to report on it, and then shifted to addressing a community of mathematicians interested in general truths, using the present tense and the general *you* voice.

Boaler would likely characterize my participant class as a traditional class. The students mostly followed and practised mathematical procedures that were given to them.

They did not generally see human agency in their mathematics, nor did they perceive themselves moving between individual mathematics and participation in a community of mathematics. Because of their traditional passive frame of reference, it was a challenge for me to draw out their *I* voices in conversation about language (and about mathematics). In this class, the teacher was seen to be the human face of the disciplinary agency, and the discipline was dominant in the dance of agency.

Joey and some of his classmates were able to overcome the typical patterns of discourse in this class and thus expressed their human agency. When they expressed their own voices and resisted the dominant voice of their teacher, they began their "dance of agency." I was trying to get them to notice that there is an appropriate use of expressive *I* and *you* voices as well as an appropriate place for generalizing voices in mathematics, but they resisted my efforts. This was the generative tension I referred to at the beginning of this chapter.

With his proclamation about the general voice, Joey displayed his awareness that he can make decisions about how to say things in mathematics class. He showed this awareness by inventing terminology, by saying that he was rejecting the *I* voice and the *you* voice (even in its general sense) and by structuring their sentences with passive and imperative voices to reflect mathematical necessity – that utterances should be generally true, independent of the perspectives of particular people.

Ironically, Joey used his display of human agency to reject human agency in mathematics. He said that he (and others) should not use the *I* voice (nor the *you* voice or the *we* voice). Had he followed his own directive, he would have cut himself off from participating in the dance of agency between his own understanding and the conventional demands of mathematical discourse. However, he did not follow his own directive. More than any other student in his class, Joey regularly exercised his *I* voice.

In the interview in which he made his proclamation about the general voice, I asked Joey if he noticed himself using *I*, *you* or *we* voices when doing his mathematics. He responded: "I'm always thinking in the *I* form when I'm doing my math. I don't know why. It's just, I've always thought that way. Because I'm always doing something."

In addition to using an *I* voice, he expressed his personal agency in mathematics by adapting his everyday vocabulary to express his mathematical ideas. For example, to describe the graph he expected from the function $y = x^4 - 2x^3 - 12x^2 + 40x - 32$, he asked Mrs. Hill, "Shouldn't there be a masher amount of hills and valleys?" I had never heard the word *masher* before, but I assumed from the context that it was roughly equivalent to the word *large*, because there would be a large number of local maxima and minima, which Mrs. Hill sometimes called *hills* and *valleys*.

My assumption that Joey saw the same mathematically significant features of the curve that I saw may be seen as an instance of what Guy Brousseau calls the *Jourdain effect*. Kang and Kilpatrick (1992) describe the effect this way: "[T]he teacher acts as if he or she recognized evidence of scientific knowledge in the students' behavior or answers, even though these responses were actually motivated by trivial causes and meanings" (p. 6). In this case, I was interpreting as a researcher rather than as a teacher, but the effect is similar. Though there is a danger of reading too much understanding and human agency into Joey's utterance, it cannot be assumed that there was no understanding or agency. As Kang and Kilpatrick (1992) assert, it is difficult to define a clear boundary between normal and pathological cases of this interpretive effect.

I suggest that the form of human agency in Joey's utterance relates to what Chapman (2003) calls *transformational freedom*: "Students with transformational freedom are those who have the facility to use language appropriate to the situation. They can express the same meanings in more mathematical or less mathematical language" (pp. 48-49). Further, Chapman argues that transformational freedom is a mark of a successful mathematics student: "[S]tudents who are successful in school mathematics display transformational freedom in their language practices, whereas students who are less successful rely on signals from the teacher to make transformational shifts" (p. 133).

Although Joey was not a high-achieving mathematics student, he showed his facility with a range of language forms and with shifts from one form to another. However, Joey does not contradict Chapman's assertions, because she claims that transformational freedom is *a* mark of success, one mark among others. This is not to say that relatively unsuccessful students cannot have transformational freedom. Joey's lack of success while demonstrating this attribute suggests that his difficulties were not due to his lack of human agency. Rather, there were other challenges for him.

Arwa and Tharshini, two of Joey's classmates who were successful in that they performed well on tests, had something to say about transformational freedom. In an interview, I told them about Chapman's thesis and asked them what they thought of it. They explained that students who want to impress and please their teacher follow their teacher's cues and use the teacher's vocabulary. Students who are confident enough of their teacher's approval do not bother with such posturing, and switch freely between following the teacher's language forms and using their own.

These two students cannot speak for all mathematics learners, but whether their account is right or not, their alternative explanation suggests some problems with Chapman's thesis. They were attending to students' *intentions*, whereas Chapman attends to the form of students' *utterances*. Chapman expresses interest in students' facility with shifts in languaging, but she does not consider the possibility that some students could possess the facility and choose not to display it. Assertions of cause or even of relevance between success and languaging practices should consider intention as well as form. Arwa and Tharshini's alternative account of the phenomenon described by Chapman serves as an example of the research benefits in talking with students about their interpretations of language practice in their classrooms.

Intention is not easily accessible. Indeed, my own intentions are rarely clear to me when I reflect on my practices. However, there are ways of approaching an understanding of intention. One way is to reflect on intention, either alone or in a group. Two days after we discussed Joey's contradiction in class, I prompted his peers to reflect on their agency by attending to a particular form of language.

I started this class by telling students I wanted them to notice when they used an *I* voice in class. At the end of class I gave each of them a piece of paper with the following prompt:

> Please write a description of an instance when you used the *I* voice in class today. It could be a time you said "I …" or thought "I…".

Their responses were depressing for me. It seems that most of Joey's classmates did not share his sense of personal agency. [11] Figure 5-10 contains transcriptions of their written responses organized according to the nature of the human agency described by the students.

relating to relationships	I should do my homework then I might actually be passing this class hahaha
	If I were you I would do my work.
	May *I* go to the washroom *I* am write you are wrong. *I* am better at math than Mr. Wagner
reflecting	Thought "I remember doing this stuff before"
	I learned how to do this last year.
	I thought about it when I was trying to solve equations.
noticing an absence	I dont remember this stuff even though I learned how to do division & remainder last year.
	I did not say anything in class today. I was never asked how to do something or describe how I got a certain answer.
	Never thought of anything where I used *I* nor say anything that used *I*.
relating to mathematics	I was explaining to a fellow student what answer I got. I stated "I got the answer *x*"
	Joey: I thought … and i quote "if i divide something by .5 it is the same as multiplying by 2, and if I multiply something by .5 it is the same as dividing it by 2." (and yes you can quote thoughts.

Figure 5-10. Student descriptions of their use of the *I* voice

Only eleven of the twenty-six students present that day gave me an answer to the prompt. Of these, only two reported using an *I* voice to express human agency in mathematics. The first grouping in Figure 5-10 suggests students expressed human agency in classroom relationships, though "May I go to the washroom?" suggests some subordination to authority because the student asked permission. The second grouping suggests that students noticed their personal agency in reflection on their mathematics learning. In the third grouping students noticed their human agency in failure – failure to remember, to speak and to notice. Significantly, the student who said, "Never thought of anything," while noticing his or her lack of an *I* voice, excluded the *I* subject from this remark. Only the two responses in the fourth grouping suggest that students noticed their human agency doing mathematics. One of these responses was Joey's.

This set of student responses illustrates a distinction between types of human agency in the mathematics classroom. Students can express human agency as a student – as in, "I will study" or "I will ignore the teacher's instructions." Students can also express human agency in mathematics – as in, "I will divide by a half" or "I will explain my reasoning." These kinds of agency can overlap too – as in "I will try organizing these examples to find a pattern."

Another student, Ricki, told me a few days later that she noticed herself thinking with the *I* voice when doing mathematics. I believe that her awareness of her human agency in mathematics is significant. First, it suggests that she was exercising her human agency, exploring her own ideas. As I have said previously, human agency is worth promoting in mathematics classrooms. Second, it suggests that she was aware of herself making decisions within the discipline; it demonstrates self-awareness. Such metacognitive functioning could help her adjust and improve her practices. Though the students in this class reported that they did not think mathematically with the *I* voice, it is possible that some of Ricki's peers also began to think more with the *I* voice.

It is conceivable that Joey's growing ability to articulate his human agency with regard to linguistic expression of his mathematics strengthened his inclination to engage in the mathematical dance of agency. It is because of this possibility that I think critical language awareness belongs in mathematics classrooms. In McLuhan's (1964) consideration of new technology, he asserts that the medium is the message. This statement can be extended to include language as a technology. If we expect students to *exercise* their personal agency in mathematics, they will need to grow accustomed to *expressing* agency in their utterances. For this to happen, they will need to hear their teachers using expressive voices in their mathematics – particularly the *I* voice – and they will need to be given tasks that give them contexts for using this voice, tasks that require personal initiative. Instead of expecting students to respond to and regurgitate their teacher's mathematics, actions that Wilder (1968) calls "symbolic reflex," we need to provide tasks that prompt students to exercise what Wilder calls "symbolic initiative." Wilder suggests that symbolic initiative is what makes us human.

Attending to the Dance Steps

Dance is about relationship. However, the relationship itself cannot be observed directly. We see only the dance steps. As we see and feel the moves, we learn something about the relationship. In mathematics, there is a dance of agency between humans and either conventionality or common necessity. This relationship expresses itself in the language that flows between people doing mathematics. If language is the dance step, then awareness of language allows us to understand the relationships between the actors in our mathematics. Though it is important to participate in the dance when we are learning it, at times there is value in attending more closely to the steps themselves.

Chapter 6 – Facing the Mathematics

> Wrapped in a blanket, with a notebook in my lap, I sit in my basement watching a videotape of a mathematics lesson. Suddenly, all the students turn to look directly at me. What has happened? Coming to my senses, I see that this is an illusion; they are not actually looking at me. This is a video record of them turning to look at something in their mathematics classroom, someone or something close to the video camera. I realize that I must have been daydreaming, because I did not notice what prompted the students to turn their faces. I reflect while rewinding the tape to watch the episode again. For this video segment to grab my attention, this kind of face-turning must be rare in this mathematics classroom. I have been watching the class for months both in person and again on videotape, but I have never noticed this before. I start to wonder what it takes for students to turn their faces in a mathematics classroom. Where do students look in mathematics class and when do they redirect their gaze?

With language, there is no direct connection between the words we use and the things to which we refer with these words. We direct attention with words, and we expect our audience to direct its attention to the things we are talking about, not to the way we are talking. We do not have immediate access to people's meanings when they speak. We only hear their words, which are symbols, and we attend with our eyes and our other senses.

What happens when we shift our attention from our words' referents to the words themselves? This chapter is about attention. One of the more generative parts of my on-going conversation with the participants in this research began with a shift in my attention. This shift was prompted by the surprise described above – the surprise I experienced when watching a video record of typical mathematics teaching.

In the first chapter, I directed attention to language as the medium of communication in mathematics (and outside mathematics). In Chapter 5, I attended to personal agency in students' mathematics communication, and made connections between a form of mathematics language practice and the nature of mathematics itself.

If language is a medium of communication, then our senses are also part of the media because we use our senses to process language. We use our ears to hear and our eyes to see people's symbols and gestures. This chapter directs attention to the bodies of mathematics students. In particular, where do they look when they talk about mathematics? What does the way they direct their gaze have to do with the mathematics they learn?

Paralinguistic Communication

Whether eye and head movements are a part of language is debatable. Lyons (1977) notes the wide range of meaning associated with the term *language*. He refers to worded expressions as "normal language-behaviour" and classifies other language behaviours as *paralinguistic*. He notes within linguistic scholarship a wide variety of ways of using this term. His definition of paralinguistic behaviour includes "both non-prosodic vocal phenomena (variations of pitch, loudness, duration, etc.) and non-vocal phenomena (eye-movements, head-nods, facial expressions, gestures, body-posture, etc.)" (p. 64). Using this definition, the way students turn their gazes in mathematics communication could be seen as paralinguistic. However, Lyons narrows his definition to include such movements "in so far as they are integrated with and further determine the structure or meaning of utterances and serve to regulate the development of a conversation and the interpersonal relations of the participants" (p. 64). With this delineation, Lyons may not consider the direction of a student's gaze to be paralinguistic. His interest seems to be in non-verbal signalling, not in signal sensing. However, I argue that listening is as much a part of language and communication as is uttering. This recognition is not new to mathematics education scholarship; Davis (1996) promotes active listening as a necessary and generative component of teaching mathematics in a complex world. The symbols or other objects at which I look when listening to a person's oral utterances inform both my understanding and my verbal responses.

It is not very important to this research whether Lyons would classify particular eye movements as paralinguistic. However, Lyons' work is instructive in drawing attention to the relationship between body positioning and verbal communication. Because verbal communication was the focus of my on-going conversation with the students in the researched classroom, it is worth asking whether our attention to body positioning – and to eye movement in particular – fits with the rest of the conversation.

After describing highlights of this part of my conversation with students, I consider some pedagogical issues that came from our dialogue. These considerations led some participant students and me to a greater awareness of the lack of access we have to each other's thoughts in mathematics communication.

In Chapter 3, I drew upon Skovsmose and Borba's (2000) framework for critical mathematics education research. They highlight the dynamic and interdependent relationships between *actual* situations and the researcher's and participants' *imagined* situations in investigations of possibility. What actually happens in such dynamic relationships sometimes defies the participants' and the researcher's imaginations. Just as I had expected, I found that conversations that I had not anticipated were frequently the most generative, both for me and for the students. The experience I describe at the beginning of this chapter initiated one of these generative chains of events that I had not imagined in preparing the research situation, for it was a departure from the kind of discourse analysis that was typical of our conversations.

The events in this chapter differ from those in the previous and subsequent chapters in two significant ways. First, this stream of conversation took a pedagogical turn. Both the principal students and Mrs. Hill addressed my prompts that directed attention to pedagogical implications of our interaction. Second, the focus of our attention was on paralinguistic features of our communication, rather than on lexico-grammatical features.

The Story – Turning Faces

Like myself, two female students in the researched class, Arwa and Tharshini, became fascinated with the way they directed their gaze in mathematics class. However, their interest in this phenomenon was sparked by a different source. My interest was piqued by my experience of a video record, while theirs came from a cartoon I showed the class.

I became interested in the direction of students' gazes when I was surprised at the video record of students turning their heads. This was ten weeks into my conversation with the students in this class. In that moment of analyzing data, I realized that it was uncommon for students to turn their heads to look at each other's faces. Perhaps the potency of this surprise was heightened by my awareness that I had not noticed this regularity despite my many years of experience with mathematics classrooms, first as a teacher and then as a researcher. In this case, it was this stark departure from the regularity that brought the regularity itself to my attention. Even though I found this realization fascinating, I did not talk with the students about my questions or observations. Regrettably, I recall thinking that my thoughts would be too complicated for them – if I, a teacher and researcher, had not noticed this phenomenon until now, how could I expect these novice students to see it as I did?

A Fascination with Pointing and Facing

Arwa and Tharshini, who were close friends, became interested in the direction of students' gazes in a different way. Their interest was piqued by part of an 800-word essay I wrote for the students in the class, entitled "What are we doing here?" (the essay appears in its entirety in Appendix B). I gave each student a copy of the essay at the beginning of the fifteenth week of the term. With this essay I wanted to draw out their responses to my sense that they found our conversations about language odd, or at least unconventional. Instead of responding to the questions I asked in the essay, Arwa and Tharshini were captivated by my use of a cartoon from "The Far Side" (Larson, 1984). In the cartoon, a man scolds his dog while pointing at a mess on the floor. The dog looks at the man's finger, not at the mess.

This cartoon resembles stories of the Buddha scolding his disciples for looking at his finger instead of the moon. He was not speaking literally about the moon. Rather, he was concerned about his disciples' dogmatic attention to his teachings, which he said pointed to truth but were not truth itself. Nhât Hanh (2002) reports that the Buddha said, "My teachings are a finger pointing to the moon. Do not get caught in thinking that the finger is the moon. It is because of the finger that you can see the moon" (p. 52).

Like the Buddha, I was not using images of physical, gestural pointing as a metaphor to represent something else – language, in my case. In the essay "What are we doing here?", I said that we typically do not think about language. We simply use it to direct attention at what is really important. I suggested that in my daily conversations about language with the students in this class, I was asking them to look at the finger instead of the things being pointed at by the finger. Instead of paying attention to mathematics itself, I was asking them to pay attention to the language with which we talked about, symbolized or otherwise pointed at the mathematics.

Arwa told me that Tharshini's pet dog was like the dog in the cartoon. It would look at Tharshini's pointing finger instead of following her gaze. This unforeseen connection to Tharshini's home prompted Arwa and Tharshini to reflect on the cartoon and on the direction of their own gazes in mathematics class. Their eyes shone brightly whenever they talked about the mysterious connection between their gazes and their linguistic communication.

In an interview two days after I gave them the essay with the cartoon, they reported that they did not look at each other when talking about mathematics. They had already told me in this interview that their goals for their classroom experience were to pass and to understand the mathematics. Arwa articulated her goals as, "to pass, and to, like, understand why, how it applies to life, sort of," and Tharshini added, "Pass. And have enough knowledge to get the grade." After hearing them describe their goals, I asked, "What does [our attention to language] do, if anything, for your goals? Like your goals for the marks and for understanding? For what it has to do with life?" They both started to answer immediately, but because of the simultaneity of their outbursts I could not understand. Arwa then indicated that Tharshini should answer first. The following conversation ensued:

Tharshini: It doesn't really do anything for my goals but like, you know the pointing thing you were talking about?

DW: Yeah.

Tharshini: From that sheet.

DW: Yeah.

Tharshini: We both realize

Arwa: Yeah.

Tharshini: When we, um, when we like help each other and stuff, we, we're aware that we point and stuff. And I don't really understand what she's talking about. I just look at whatever she's pointing at.

Arwa: And, like, it helps us, sort of, helps us visualize like, what you actually want us to know. It's like with the language talk and stuff. Because it gives us a sense about how you want us to answer the questions. And not just like, "yeah this is the answer," but explain why, and how, like the pointing picture thing. That was one of the things, that we like didn't realize but when people do look at something, we actually look at it, and we're listening but we're sort of visualizing where they're pointing.

Tharshini: Yeah. You don't look at the other person.

Arwa: Yeah. It's like that cartoon.

Tharshini: It's more like imagining what they're pointing at, trying to understand what the diagram's about.

DW: So, even if they're not pointing with their finger, but just talking, you just listen and imagine what it is they're talking about.

Tharshini: Yeah.

Arwa: Yeah. But if they point, you just sort of

Tharshini: If, you're more directed towards it?
Arwa: Yeah.

Transcript 6-1. Arwa and Tharshini reveal their interest in the direction of the gaze

Tharshini noted here that a mathematical diagram is important because it directs attention to something more important: "[T]rying to understand what the diagram is *about*." They continued, saying that they also noticed the phenomenon in their classmates' conversations:

DW: Yeah. That's interesting. Because it's something that I've noticed too since being in this class with you guys. Because one thing that I notice is that when people talk about the math, even though they're arguing, where do you think people look? Like, let's say, in the class discussion, Tharshini disagrees with Arwa. Do you think she'd be looking at Arwa?
Tharshini: Yeah, we'd be looking at the diagram for sure. Because even when like Joey and Kyle and stuff when they argue, or Brandon, they're always like pointing at the overhead. They'd be like, "Over here, can't you see it" and stuff.
DW: Yeah. And the others don't look at the person. | They just look at |
Tharshini: | Yeah |
Arwa: | Yeah, they don't |
DW: Yeah.
Arwa: | It's not like they |
Tharshini: | Because we don't see what they | see or anything
Arwa: Yeah, because what they're seeing. We can't really see through their, um, their eyes. But, like, when they point we sort of visualize actually what they're looking at too, because of the focus to look at.
Tharshini: Yeah.
DW: Yeah, I find that very interesting too. So, yeah. Okay. So, anything

else?

Tharshini: That's pretty much it.

Arwa: Yeah.

Transcript 6-2. Arwa and Tharshini noticing the role of symbols

This conversation was such a pleasure for me. Its tone is lost in transcription, but the numerous times that we said "Yeah" testify to our mutual sense of awe that we were noticing the same phenomenon. This word appears in other transcripts I include in this dissertation, but nowhere is it even nearly as frequent as it is here. Each "Yeah" felt like a "Yes, that's it. That's exactly it. We are seeing the exact same thing." Ironically, we were taking pleasure in our sense that we were seeing the same thing, when in fact we were seeing that it is impossible for us to see the same thing. Arwa and Tharshini each articulated well the impossibility of seeing the same thing – Tharshini said, "[W]e don't see what they see," and Arwa repeated, "We can't really see through their [...] eyes."

This irony also pervades my account of the conversation. In this chapter, I often write about Arwa and Tharshini as though they were of one mind, because they regularly interrupted and continued each other's utterances. With the fluid connectivity of their utterances, it was often difficult to distinguish between their different points of view. They were good friends and reported to me a few times their fascination with the transcripts of our discussions, because these transcripts seemed to support their belief that they thought the same thoughts as each other. Ironically, their observations about language use and pointing reveal their awareness of the problematic nature of this belief. They spoke as though they knew what the other person was thinking even as they articulated their awareness that each could not possibly know what the other person was thinking.

During the conversation represented above, these girls' observations surprised me because they seemed to be describing the same thing I had noticed a few weeks earlier. However, they added a new interpretation. They noticed what I had noticed, that students looked at the mathematical symbols instead of at each other. They added that even though students appear to be looking at the same thing, they would not be seeing the same thing.

A still photograph taken from video data shows an example of Arwa and Tharshini interacting in the way they described (Figure 6-1). In the image, Arwa leans across the aisle to talk mathematics with Tharshini. Arwa and Tharshini both look down at the symbols on Tharshini's paper as they talk.

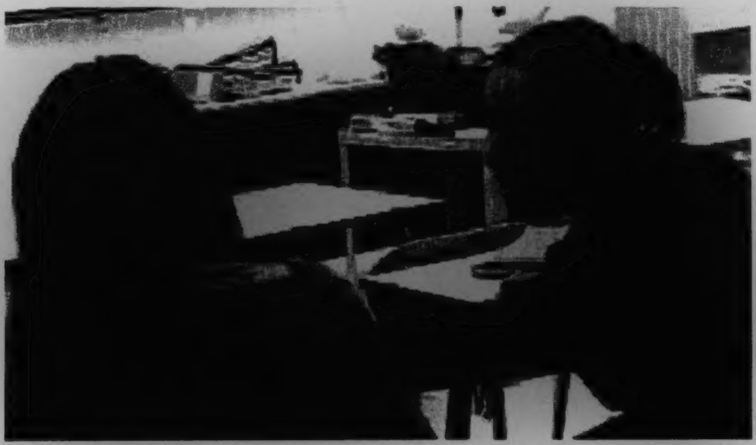


Figure 6-1. Arwa and Tharshini facing mathematics

A still photograph taken by Gordon Calvert (2001) provides a clearer image of two people looking at their symbols instead of at each other as they do mathematics together (p. 95). In her picture, Ken and Stacey appear to be looking at their hands or at the piece of paper with which they are both working. They lean forward, Ken with both arms on the table, his right hand working with a pencil on the paper and his left hand nearby. Stacey's shoulder rests on the back of Ken's right arm as her right elbow is on the table. The activity of her gesturing hand seems to be drawing their attention.

Gordon Calvert (2001) describes the intimacy Stacey and Ken brought from outside their mathematics to their mathematics and wonders about a sense of intimacy that accompanies any mathematical interaction:

> Perhaps doing mathematics with another, particularly in conversation, requires some level of intimacy. [...] A mathematical conversation requires that individuals work closely, within the same space, rather than across the table with a barrier between their bodies. Being too far removed from this interactive and interpersonal space, and the communicative tools of mathematics perhaps precludes the possibility for a collaborative relationship in mathematics. The mathematical space is necessarily close, intimate. (p. 95)

Gordon Calvert (2001) describes the sense of intimacy she observed as an act of love, using a characterization of love from Maturana and Varela (1987). In quoting them, she says "love lets us *see the other* person and open up for him room for existence beside us" (Gordon Calvert, 2001, p. 96, emphasis mine). However, Stacey and Ken were not *looking at* each other while they were *seeing* each other. Their mathematical symbols and the gestures of their hands seem to have been necessary intermediaries in their relationship, as they saw each other through the symbols they shared. As they engage in mathematics conversation, it seems that people can see each other's mathematics best when they see through their symbols.

Ironically, the intimacy engendered by shared mathematics does not seem to be represented in the form of the communication. As I showed in the previous chapter, there is a tendency to mask human agency in the structuring of linguistic utterances. And as Arwa, Tharshini and I have noticed, there is a tendency to accompany linguistic utterances with face-avoiding paralinguistic gestures.

Balacheff (1988), in his analysis of proof practices in school mathematics, also provides images of students doing mathematics together. There is a 35-frame cartoon in his appendix showing two children working on one of the tasks that are the bases of the student work that is analyzed in the chapter. The cartoon seems to be a unique medium for presenting a transcript of mathematical conversation. For someone with an awareness of eye movement, there appear to be oddities in the cartoon. The children seem to be looking at each other instead of the symbols that they are talking about. For example, in the fifth frame (p. 231), Bert says, "don't make it concave […]. Look … it's not the same," while looking at Chris, who is drawing and holding his paper in such a way that Bert cannot see his drawing. In the twenty-second frame (p. 233), Bert talks about his drawing and points at it while looking at Chris. In the twenty-fifth frame (p. 233), Bert and Chris look at each other as they talk and "think" a diagram. (Only in cartoons can we actually see a person's thoughts.) They are "seeing" the diagram, though their eyes are not looking at it. During the entire conversation, the children's eyes never look together at their symbols. This cartoon might appear to contradict the finding that I shared with Arwa and Tharshini. However, I argue that the oddities I point out suggest a problem with Balacheff's highly interpretive images. I suspect that when Bert and Chris were doing the mathematics depicted in the cartoon, they did not look at each other as much as the cartoon suggests.

Gordon Calvert's (2001) consideration of larger groups than two supports the idea that participants in a mathematical conversation need to share their symbols:

> In [groups of three] one person, the one sitting on the end or on the other side of the table frequently could not participate fully simply because he or she could not see the mathematical references put on paper. Communication of mathematical ideas is in large measure nonverbal in terms of its reference to symbols, diagrams, and drawings on the page, as well as the gestures which include pointing to, as well as talking about. (p. 96)

In larger groups yet, such as a whole mathematics class, conversation is usually mediated through writing and other symbols on a blackboard, or displayed using an overhead projector. It is more difficult to see where students look if the teacher is close to the mathematical symbols. Do students look at the symbols or at the teacher? Arwa, Tharshini and I have noticed that when students in their class argued and conversed with each other about mathematics, they did not look at each other. They continued to look at the symbols displayed at the front of the room.

It is said that the eyes are the windows to the soul. We may look into the other person's eyes to see the soul, but it appears that we need to look at the symbols, including symbolic artefacts of the person's written utterances, in order to face their mathematics.

Though I had already noticed that the people in this class looked at symbols instead of at each other when they conversed about mathematics, Arwa and Tharshini's similar observation prompted me to reflect again on what I had noticed. Their sense that this feature of communication was connected to the impossibility of two people actually seeing the same thing when looking at the same symbols added depth to my reflection. This nuance suggests that they were aware that the symbol is not the mathematics – it is a path to the mathematics.

With the combination of their two observations, that mathematics is seen through symbols and that no two people can see any symbol in the same way, Arwa and Tharshini seem to share Duval's (1999) sense of the inaccessibility of mathematics. I was particularly pleased with Arwa's and Tharshini's insight, because in my preparation for this research I had predicted in a conference paper that students engaging in discourse analysis and attending to their language practice would be more likely to notice this peculiarity of mathematics discourse:

> Because of their intimate acquaintance with its context, participants in a language-aware mathematics classroom ought to have an easier time of being aware of semiotic tensions in language when they analyze their own discourse. […] Awareness of this semiotic tension might help students understand how, as Duval (1999) puts it, "there is no direct access to mathematical objects but only to their representations." (Wagner, 2003b, p. 361)

Testing Observations about Facing Mathematics

The day after the interview with Arwa and Tharshini (transcript excerpts above), I reported on it to the class. Their classmates agreed with the observation that they did not look at each other when talking about mathematics. After saying that Arwa and Tharshini were interested in where people look during mathematics conversations, I asked the class, "Where do you look when you're talking about mathematics with someone? Do you look at the person? Or, do you look at the page that you're working on?" Most of the students responded immediately, saying, "[T]he page." Everyone who spoke seemed to agree, but at this point no one said more than an expression of simple agreement.

The next day, I again brought the topic up for discussion. This time, the students had a class period to think about the question and to pay attention to their own practices after I asked them the question. Apparently, Rory was the only student to take interest in the question; she was the only one who talked during this discussion about language.

DW: At the beginning of yesterday I pointed out that um Tharshini noticed, that she told me that she noticed that when you talk about math with someone, you don't look at each other, you look at the thing that you're talking about. Did you notice that yesterday? Did anyone pay attention?

Rory: It's true.

DW: Yeah? It's true?

Rory: Yeah. Because sometimes you try to look at the person, because you should be looking at them when they're talking.

DW: Then what happens?

Rory: Well, then you realize that you're not learning as much.

DW: Oh, okay. So, the person being who? Your teacher?

Rory: | Yes. | [*simultaneous*]
DW: Or, | your friend? Or |

Rory: Anybody.

DW: Anybody. Oh, okay. So you realize that you don't understand it as well when you look at the person?

Rory: Yeah.

DW: Anyone else notice? Anything?

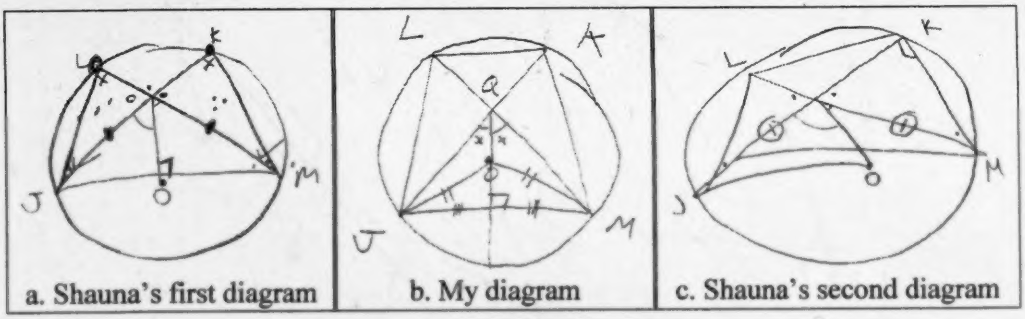
[*silence*]

Transcript 6-3. Rory experiments with directing her gaze

During this conversation, Rory experimented with directing her gaze in different ways. Though she was speaking with a generalizing *you* voice ("you try to look," "you should be looking" and "you realize that you're not learning") I had the sense that she was talking about her own experience. When I said "your friend,", she looked at her friend Kalli (I am basing this observation on memory, because Rory and Kalli were outside the video camera's range). She was describing her experimentation, which came from her own initiative. How could she expect that other people did the same kind of experimentation? And if they did, how would she have known what they noticed?

Rory found that she had trouble understanding when she was not looking at the symbols being discussed. Her reflection prompted me to think about effective communication practices for mathematics educators. When speaking mathematics, if we expect students to write notes *and* understand, we should enforce enough pauses so that students can write without missing out on listening – the kind of listening that requires looking at the writing and symbols being referred to by the speaker.

After the extended silence in the above dialogue between me and Rory, I gave a counterexample. I told the class of an argument I had had with Shauna about a particular proof. I had suggested to the class that this proof was far more difficult than it would seem. When the students were working on proofs independently and in small groups, Shauna called me over to explain her proof. As she explained, I was looking at her first diagram, which she wrote on my scrap paper (see Figure 6-2a). I thought she was overlooking a key assumption that she was making and I pointed to my diagram (Figure 6-2b). She countered with a second diagram to direct attention to her argument (Figure 6-2c). The many dots on these diagrams, only some of which appear clearly in these scans, are artefacts left by our often forceful pointing, me with my pen and Shauna with her pencil.



a. Shauna's first diagram | b. My diagram | c. Shauna's second diagram

Figure 6-2. Proof argument jottings

All three of these diagrams were on the piece of scrap paper that I had been carrying around in my notebook. Though the diagrams were in close proximity to each other, she noticed when I looked away from her second diagram toward my own diagram. This perception was rather startling, for she appeared to be attending to her own diagram. How did she know I had turned my gaze elsewhere? I did not notice myself shifting my attention until she pointed it out and scolded me for not looking at her diagram, her pointing and her gesturing. I responded by saying that I could listen to her proof better while looking at my own diagram. (This conversation was not recorded.)

After I gave the class this counterexample, in which it helped me to look at my own diagram instead of Shauna's, no student said anything more. On the next day, I brought up the same question again, but focused on pointing and gesturing. Some students made a joke out of the discussion and effectively terminated the dialogue. I talked with Arwa and Tharshini about their observations one more time, in the last week of the term. I will refer to their comments later in this chapter.

Symbols and Mathematics

I suggest that the reasons why we look at people's symbols instead of at the people themselves relate to properties of mathematics itself. In Chapter 5, I made a similar suggestion, saying that the neutral voice in typical mathematical utterances points to a discipline that is abstract and general, favouring conventionality. Now I ask what it is in mathematics that might direct students and others to communicate in the particular way that Arwa and Tharshini described for me. Perhaps the tendency not to look at the person in favour of the symbols is another aspect of the linguistic manifestation of the abstract, general nature of mathematics. In Chapter 5, I focused on the way we draw attention away from individual people by favouring non-personal voices in our sentence structuring. Here, I focus on the way our attention is drawn away from the conversants' faces themselves, as we prefer to look at the symbols instead.

Though I am fascinated by the connections between these two streams of conversation – human agency in the domain of languaging mathematics and the direction of our gazes in the domain of mathematics – I will direct attention to a different feature of mathematics that is informed by the paralinguistic gestures I discussed with Arwa and Tharshini, and briefly with Rory – the inaccessibility of mathematics and the significance of symbols in this discourse that provides no immediate access to the objects of conversation. We look at symbols when we do not have access to the objects to which the symbols refer.

In an earlier conversation with the whole class, I had tried to draw attention to the relationship between symbols and the objects to which they are intended to refer. While introducing this day's conversation about language, I told the class that the study of this relationship is called *semiotics*, but I did not use the word again in our conversation. Because I had brought into this conversation an interest in semiotics in the mathematics classroom, I was disappointed that the students seemed relatively uninterested. Neither Arwa nor Tharshini showed any interest in this discussion. A few students engaged in conversation as I led it, but no one pursued the conversation with particular interest. This conversation happened in the tenth week, five weeks before my initial conversation about pointing with Arwa and Tharshini.

As a departure from routine, one day I introduced explicit discussion about language in the middle of a lesson. After informing the class that I wanted to direct their attention to language, I drew the following image on the overhead transparency (See Figure 6-3) and asked, "What's that?"

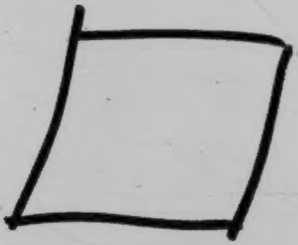


Figure 6-3. "What's that?"

The following transcript represents the dialogue that ensued:

Signot: Square.
girl: It's a square.
DW: It's a square?
Joey: Well, it's got a little dealy on it so it's not really.
DW: How do you know it's a square?
Joey: It isn't.
Signot: It looks like one. It's a figure.
DW: It's a figure?

Joey: Because it doesn't have those little things that show that the sides are symmetrical.

DW: The thing is, that if I were to do stuff, if I did a question that had to do with squares and I drew this picture you'd all think of it as a square, right? It's, it's a representation of a square. Is there such a thing as a perfect square in this world?

Transcript 6-4. Looking at a "square"

The majority of the class seemed to recognize the figure as a square, though it was significantly messy. Only Joey disagreed, drawing attention to the "little dealy" on the top left corner. As the conversation continued, I argued that there is no such thing as a perfect square in the world. I asked for the students to provide real-world examples. For each example they gave, I showed how it was not a perfect square by drawing attention to lines that are not perfectly straight or perfectly parallel, angles that are not perfect and vertices that are rounded. I even asked Mrs. Hill if she knew of any perfect squares. The following transcript picks up the conversation as it continued from this point.

DW: Do you know of any perfect squares in the world, Mrs. Hill?

Mrs. Hill: Oh, no. I don't know. [*laughing*]

Signot: What about the one you did there?

DW: This one here? [*indicating the one just drawn, Figure 6-3*]

Jessye: No, the one that was an odd number squared. [*referring to another square, Figure 6-4*] Perfect to that, I guess.

Transcript 6-5. Are there perfect squares in the world?

I am not sure to which square (or image) Signot was pointing when he said "the one you did there," but I am interested in Jessye's observation when she directed attention to "the one that was an odd number squared." She seemed to be referring to the "perfect square" I had referred to earlier in the class. Figure 6-4 is a copy of the words and other symbols I used when demonstrating a visual proof of a conjecture: the sum of consecutive odd numbers starting at one is a perfect square.

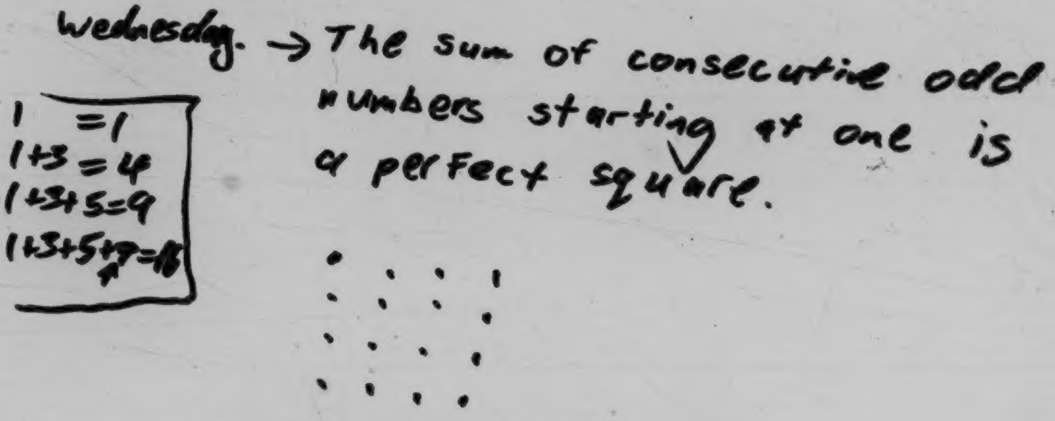

Wednesday. → The sum of consecutive odd numbers starting at one is a perfect square.
1 =1
1+3=4
1+3+5=9
1+3+5+7=16

Figure 6-4. Symbols used in a visual proof

These symbols in themselves represent very little of the communication. In particular, the four by four grid of dots did not appear all at once. I had begun with one dot, talked about adding three more to make the next perfect square, adding five more for the next perfect square and adding seven to make the last one. I had indicated that the pattern could continue indefinitely.

In Pimm's (1995) discussion of static and dynamic imaging in mathematics, he notes the added meaning contributed by the gestures used to form an object:

> We regularly talk about *drawing* a diagram, but also about *drawings*. The word 'drawing' suggests the process rather than a result (that which is drawn), and drawings necessarily evolve in

> time. Watching how students actually draw diagrams can inform our understanding of how they are viewing the problem. (p. 54, emphasis his)

Nunokawa (1994) also makes a distinction between the "ways of drawing" and "the results of drawing." After careful consideration of a student's drawing processes and the completed drawings, Nunokawa concludes, "the way of drawing the diagrams reflects the sense of the diagrams and can complement the meaning which is displayed by the diagrams themselves" (p. 37). I would add that the way of drawing can also complement the meaning of verbal utterances that accompany the process of drawing. In this way, with the drawing's supportive role in meaning, Lyons (1977) would grant these gestures the status of being paralinguistic.

At the time of her observation, I dismissed Jessye's observation. On reflection, I realized that she might have been pointing at the heart of the problem I was trying to lay bare. I was talking about physical, concrete perfect squares, which I still believe to be non-existent, but she was talking about the *idea* of a perfect square. The idea of a perfect square does exist. This idea can be represented in geometry, and it is also represented in the definition of a square number. *One* is a perfect square, *four* is a perfect square, and *nine* is a perfect square. Though these symbols are said to be perfect squares, there is nothing square about them except that they refer to numbers of like objects that may be arranged to form perfect squares – not concrete perfect squares, but representations of the ideal perfect square. In the moment of the classroom discussion about perfect squares, the thought of square numbers did not even cross my mind.

After Jessye's observation I showed the class a copy of the René Magritte painting "Ceci n'est pas une pipe" – the painting that Pimm (1995) uses to differentiate between symbols and the objects to which they refer. (I briefly described his use of the painting in Chapter 2.) Our conversation continued, now with the Magritte image before us:

DW:	This is a famous painting by René Magritte.
Signot:	What's it say.
DW:	Someone who knows French, translate it for us.
Raina:	This is not a pipe.
DW:	This is not a pipe? What is it then?
Signot:	Pipe.
Joey:	It's a drawing of a pipe, not really a pipe

Transcript 6-6. Conversation about linguistic representation.

Though Joey showed that he was aware of the distinction between the pipe and its representation (and his classmates seemed to understand and agree with him), the class did not seem very interested in the distinction. It seemed to be only Arwa and Tharshini who took up this distinction in their response to the "Far Side" cartoon. It is not possible to know the extent to which our class conversation about semiotics had an influence on these two girls. Even they themselves could not know.

Where Mathematics Is – "The Other Side"

I am interested in two somewhat related aspects of the way students turned their gazes during mathematics discussion. I noted above the connection in mathematics communication between students turning their gazes away from their interlocutor's face and the linguistic structuring that masks the person's human agency. Now I will explore how we use language to express the physical distance between our mathematics and ourselves.

It is significantly different talking about a pipe, which my conversation partners and I can all experience, than talking about a square, a parabola or a proof, which only exist somehow in our imaginations. Yet I note the tendency to speak of these inaccessible things as though they are present, like a pipe that is in our hands. In English, mathematical utterances are often characterized by the present tense and by pointing words like *here* and *this*. In Chapter 3, I referred to Levinson (1983) and Wagner (2003b) to compare proximal deixis – present tense and pointers *here* and *this* – with distal pointing – past tense and the words *there* and *that*.

My limited knowledge of another language helps me understand the apparent oddity of our practice of referring to distant things as though they were proximate. In siSwati, there are three words for referring to the position of objects – *lapha (LAH-pah)*, *lapho (lah-POH)* and ***lapha*** *(lah-PAH)***.** Notice the different stress in the pronunciation of this last version, despite the fact that it is spelled the same as the first one. "*Lapha*" means "here." "*Lapho*" means "there." And "***lapha***" is often translated as "on the other side." It refers to things beyond our view. I find it interesting that the words for "here" and "way over there" are the same word with a mere difference in emphasis – lapha and la**pha** – especially since the emphasis of syllables is usually unimportant in this language. If I say "*this* pen, here in my hand," I would use *lapha*. If I say "*that* table" and I am pointing at it so that my audience can see it, I would use the word *lapho*. If I am outdoors and talking about a piece of paper on my desk in my office, I would use the word ***lapha***. It is far away, beyond our view.

In English there are similar practices. We talk about far away or abstract things as if they are proximate by using the present tense and pointing words like *this* and *here*.

The alternatives – using the past tense and the words *that* and *there* – suggest distance. When I talk with a friend about a piece of paper that is far away on my desk and describe how I can fold it to achieve a certain result, I display a sense of proximity with my pointing words when I say "this paper." I also indicate proximity by using the present tense: "I fold the top right corner to the centre line." These common ways of speaking suggest that the paper is right here in my hand, where my audience can see it. Yet in reality it is far away – la**pha**. Indeed, it might not even exist. Though it might be a hypothetical piece of paper, it does exist in my thinking and in my audience's thinking. Because we see this paper only in our imaginations, it feels even more proximate than the sidewalk we are standing on. However, we each imagine our own "piece of paper," and we cannot see each other's papers. We talk as though we see the same paper, but our words that point at and refer to this mythical piece of paper actually refer to more than one piece of paper. Each person refers to her or his own imagined piece of paper.

Tharshini gave an example of this oddity. She described conversations she had had with another friend who was taking the same course – grade 11 Pure Mathematics – but was not in the same class. When they talked about their mathematics learning, they spoke as though they had experienced the same lessons even though they knew that their lessons were significantly different. Tharshini said that when she would talk about a lesson she had just experienced, her friend "wouldn't really be focusing on what I'm saying, but [...] trying to remember back whatever *they* did that day, and then try to understand."

Tharshini is not the first person to notice our human tendency to project our points of view onto others when we interpret their words and actions. Sartre (1956/1992) illustrates this phenomenon in his phenomenological account of "the look." He uses a description of himself watching an anonymous man in a public park. He refers to this person as "the Other" because he could be any other person. The way Sartre sees the man in the park is significantly different from the way he sees the lawn along which this man walks. The lawn is inanimate. Because the "Other" is a man, Sartre projects himself into the Other's position as a way of interpreting the Other's actions:

> this green grass [...] exists for the Other [...]. This green turns toward the Other a face which escapes me. I apprehend the relation of the green to the Other as an objective relation, but I can not apprehend the green *as* it appears to the Other.
> (p. 343, emphasis in original)

Sartre recognizes the impossibility of seeing what another person sees. Yet, in the face of this impossibility, he is aware that he cannot resist thinking that he does know what the other sees.

In the dialogue in this class, I found examples of both proximal and distal pointing. During my interactions with students, I did not think to discuss the differences between these kinds of deixis, but I think such a conversation could have been generative. I would like to know what sense students make of their distinctions between the pointers *this* and *that*, for example.

The purpose of my research is not to describe classroom practice. Rather, I aim to uncover a range of possibilities for classroom dialogue. With this goal in mind, it is sufficient to show some examples of the way distal and proximal deixis were used in this classroom and to ask questions about the distinction. In any particular case, one cannot be sure of the meaning. Indeed, the meaning and effect of an utterance and of a particular word in the utterance are different for each of the thirty different people in a class.

For example, in the following excerpt from a conversation about the distance between a line and a point, both defined on the Cartesian plane, the words *this* and *that* both appear:

DW:	In this case we'd have four *x* minus twenty. So *x* is four and *y* is negative two, right? So the intersection is at
Joey:	four, negative two.
DW:	Four, negative two.
Kyle:	To solve that, can't you just make the bottom one negative and then solve that?
DW:	Oh yeah, you could subtract, yeah. Okay, so we've got that point. [*pause*] We've got this point and this point. Can we find the length?

Transcript 6-7. An example of distal pointing

In this excerpt, and in the conversation that surrounded it, I used proximal pointing to refer to the tasks I posed at the front of the class and to the responses I recorded on behalf of the students. I said, "In *this* case" instead of "In that case." When Kyle asked a question about what I thought of as *our* mathematics, he used distal pointing: "To solve *that*" instead of "To solve *this*." What does this mean? Does it mean that I felt ownership of the mathematics and Kyle did not? When Kyle said, "can't you just make …," he did show some ownership of the mathematics by displaying his human agency, suggesting an alternative approach to the one sanctioned by me, the voice of disciplinary agency. However, even in this contribution, he deferred to my authority by asking for permission or confirmation – asking, "Can't you?" Though the situation is complex, the consideration of these pointing words draws attention to a significant issue in mathematics class dialogue: To what extent do the various participants feel ownership of the mathematics?

After this initial exchange, I responded to Kyle and used both pointers – *this* and *that*. I cannot recall my intentions at the time of the utterances, but I can say what I suppose might have been going on in my thinking. This conjecture can account for my switch from distal to proximal pointing. When I said, "we've got *that* point," I seem to have been thinking that we had completed the task. I was looking back over our work and directing attention to the point, which I thought was the desired answer. The distance I suggested with the pointer *that* is similar to the distance that I felt was being suggested by a student in my master's research when she referred back to a mathematical investigation from a week earlier: "We needed more time on *that* project"(Wagner 2003b, p. 358, emphasis not in original). After my exchange with Kyle, I repeated myself with proximal pointing – "we've got *this* point and *this* point" – after realizing that we had more to do. These two points were useful for our primary goal, but not the final answers we sought. It appears plausible to conclude that I used proximal pointing while I was immersed in the question and distal pointing from outside the question.

This wording of this interpretation does not seem odd, but questions can be asked to make it appear odd. Is it I in the question, or the question in me? I have asked a similar question about mathematical problems (Wagner, 2003c): "I cannot be *given* a problem. I cannot even *have* a problem in the sense that I have control over it. Rather, my problems hold me – they make me captive" (p. 612). The sense of possession illustrated in my brief analysis of the exchange with Kyle is not a simple thing. When we sense ourselves to be in close relation with someone or something, we can find the boundaries blurred between self and other.

A Pedagogical Turn

We talk as though we experience the same thing, even if it is a piece of paper on the other side of the world. That is where mathematics is. It is la**pha**, beyond our senses, but we talk about it as if it is here. I sense that this gulf between the actual location of mathematical objects (inaccessible) and the apparent location (tangible) suggested by the way we speak of them is one reason why communication in mathematics is complex and often frustrating.

I believe that if students become aware of this problem, they could feel better about being confused. Indeed, I think confusion should be expected. When I was in grade school, my teachers told me that mathematics was precise, unambiguous. I was successful in mathematics, but I am concerned about the students who struggled. How did they feel about not seeing inaccessible things that were supposed to be so clear? How did they feel about not seeing things that were talked about as though they were present and at hand? I think they had good reason to be confused. I am not suggesting that we change the way we talk about mathematics to make it seem less proximate. Instead, I suggest that there is value in helping students become aware of the reasons behind the semiotic tensions related to speaking mathematically.

It may be significant that some students in the researched classroom I have described summed up their sense of the value of attending to language by saying that it made them more comfortable in class. The students who noted this benefit did not know why they felt this way, but they felt a new sense of security, a feeling that they had not experienced in their previous mathematics classes. It seems ironic that their awareness of the complex relationship between symbols and meaning in mathematics would make them more comfortable, but I suggest that it can be comforting to know that our confusion (or fear) is appropriate.

Metaphors for Understanding the Role of Symbols in Mathematics

My knowledge of siSwati pointing words has helped me understand the tensions inherent in communication about inaccessible things such as mathematics. After talking with Arwa and Tharshini about the direction of students' gazes as they communicate mathematics, I tried to envision some other way of making this tension clear to students. What metaphors would be accessible to them?

In my last interview with these girls, they made a comparison that might be accessible to most mathematics students. When their history teachers would point at maps to talk about Napoleon's invasions of neighbouring countries, for example, students were supposed to be thinking about the wars – the fighting, the movement of fighting men through populated and rugged terrains, the social issues that prompted the fighting and the social issues that were caused by the fighting. Sometimes, when their teachers pointed at maps while talking about the Napoleonic wars, Arwa and Tharshini would find themselves thinking about the wars as though they were transformations of shapes on the map (as the boundaries changed with the moving fronts). They knew that they were supposed to be looking *through* the map instead of looking *at* the shapes on the map. Maps are symbols that represent something that cannot be seen in a classroom. Students who are unaware of the representative role of their maps miss the point of their study of history.

Similarly, mathematics students who see themselves manipulating symbols, unaware that these symbols are representations of something else, miss significant aspects of mathematics. Mathematical symbols, whether they are algebraic, geometric or words spoken to refer to mathematical objects, should be looked through, not looked at. They should be transparent, like windows.

In the Chapter 5, I described Adler's (2001) dilemma of transparency. Adler draws on Lave and Wenger (1991), who suggest that in fluent practice, resources are used as though they are transparent. She extends their image by including language as a resource. I suggest that mathematical symbols are also a resource students have, though not all students have the same facility with the resource. Symbols need to be used as though they are transparent. They need to be looked *through*, not looked *at*. However, as I said regarding the transparency of language, there is value in occasionally looking at the symbols instead of only looking through them. Indeed, this is what Arwa and Tharshini were doing when they talked with me about the role of symbols in mathematics communication. I suggest that this time of looking at the symbols while looking through them and of watching peers look through symbols afforded these two girls an opportunity to understand their mathematical processes better.

Another metaphor for the way we imagine seeing each other's meaning can be found in many high school mathematics classrooms. I think of times when students and their teacher use electronic graphing technologies. All students look at their individual screens, but they talk as though they were all seeing the same image. In fact, because different students have different graph window settings, students see significantly different representations of any given function. This is similar to students looking at a common symbol while discussing mathematics except that, in this case, instead of gazing together at a common symbol, each student looks at his or her own calculator image and imagines it is the same as his or her peers' images.

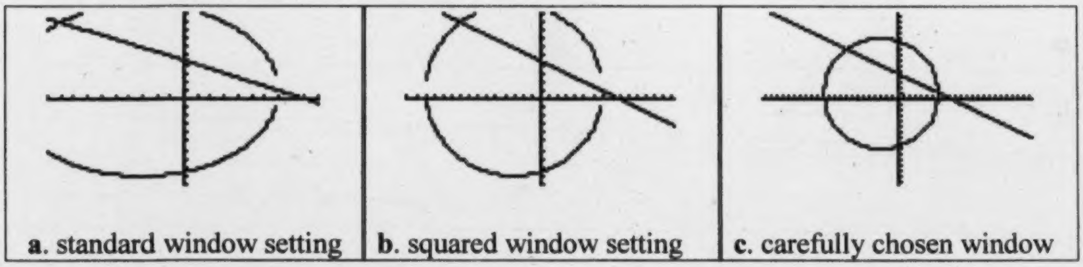
This discrepancy mirrors Arwa's and Tharshini's recognition that students in mathematics class do not look at each other, and that they speak as though they know what the other people see in the mathematical symbols. Although we cannot see what others see in the symbols, our images will be similar enough that it usually makes sense to assume we are seeing the same thing, as with the graphing calculator images.

Conversations about calculator windows can exemplify how easy it is for participants in conversations to assume they see the same thing and how frustrating a conversation can become when this assumption is false. In one class, Mrs. Hill had the students considering a polynomial function. She said, "[F]irst we'll graph it." Most students dutifully picked up their graphing calculators as Mrs. Hill picked up hers. After they had all had sufficient time to punch in the function and look at its graphical representation, Mrs. Hill asked, "Can someone find the relative maximum here? Neeta?" Though Mrs. Hill was using proximal deixis with her use of the word *here*, Neeta looked at her own calculator to answer the question. There seemed to be nothing out of the ordinary, because they assumed they both saw the same image. Indeed, Mrs. Hill tended to ask the class to use the same graph window settings as each other when they graphed the same function. Usually she asked for a student to suggest a good setting. When they all used the same settings, it was easier to presume that all students would see the same image; they all had the same kind of calculator. However, students did not always use the conventional window settings.

Another conversation about calculator windows exemplifies the importance of remembering that we do not really see the same thing in our conversations. In this case I was teaching and I had asked the class to find the points of intersection of the line $x + 2y = 9$ and the circle $(x + 3)^2 + (y - 1)^2 = 100$. The students picked up their calculators to graph the line and the two semi-circles $y = 1 \pm \sqrt{100 - (x+3)^2}$ that would make up the circle relation. After giving them time to enter the functions and look at their graphical representations, I asked the students to describe the circle for me. Chaos ensued because most students' circles did not look like circles. Figure 6-5 shows three different images derived from the line and circle relations above. The images come from the same calculator but have different "window" settings.

In the class discussion, some students' circles looked elliptical, as in Figure 6-5a, because they used a "standard" window setting. The misshapenness is a result of different scaling on the x- and y-axes. Also, most students had gaps in their "circles," as in Figures 6-5a and 6-5b, because the calculators graphed using tables of values and missed the points at the extremes of the relation's domain. Figure 6b is the result of using squared scaling, but the gap remains. For the students with gaps in their circles, the line passed through the gap. Figure 6-5c shows the full circle with no gaps and a circular shape. This result would require of a student either some trial and error or exceptional knowledge of circle relations and of the peculiarities of the calculator.

The students already knew that something was amiss. They were turning their heads to look at each other's graphs. Brandon said, "I have a circle." His response to my prompt suggests that he was aware that most of his peers did not have circles. Kyle said, "Me and Gary don't." The students were aware that their usual assumption that they all see the same thing in their calculator windows was not viable. Their language choices reflected this awareness. They did not speak as though they shared a common image. They differentiated between individuals' images.



a. standard window setting **b**. squared window setting **c**. carefully chosen window

Figure 6-5. Three different calculator images of the same pair of relations

Pedagogical Ignorance

The metaphor of the graphing calculator windows relates to a mathematics teaching strategy I developed when I taught high school mathematics. Here, I will describe this strategy and then work backwards through the metaphor to apply the strategy to classroom mathematics communication.

In the 1990s, when students began using graphing calculators in my mathematics classes, I noticed in our discussions that their descriptions of their graphs tended to be quite shallow. For example, if I asked my students to describe the graph of $f(x) = x^2 - 4x$ so that I could sketch it on the blackboard, students would have their calculators sketch the graph and one student would say something like, "It's down and to the left." Such vague responses, with unclear deixis and insufficient detail, left me to fill in and assume the student's awareness of the details. The students would expect me to sketch the graph on the blackboard despite their vague descriptions. I realized over time that the weakness of their descriptions might be related to the students' awareness that I was looking at the image on my own calculator, which they assumed to be the same image they saw on theirs. My students felt that I need only copy the image from my calculator. My question to them, when I asked for a description of the graph, seemed like a mere test of their knowledge or their ability to graph using their calculators.

To draw on and exercise the students' communicative competence, I decided to stop holding a calculator when I taught. Students' explanations improved immediately. If I was not holding a calculator, a student answering the same question as above felt compelled to tell me more than the simple "down and left," because I could not see the graphed image. The student had to look for important markers, like the places where the curve crossed the *x*- and *y*-axes, or the place where the graph dipped the lowest. Their search for appropriate reference points would direct their attention to zeros and to the vertex. Though I could "see" in my imagination the graph with its zeros and vertices, the students could not see me seeing it. Because they saw me looking at them instead of at a computed image, they saw my question as a genuine question, a question that they would struggle to answer well. They saw themselves as knowing something that I wanted to know.

Working backwards through the metaphor, in which I compare graphing calculator images to students' and teachers' private images of the mathematics they see represented by symbols, I ask how a teacher might do the same trick I did when I discarded my calculator. In everyday classroom conversations about mathematical ideas and processes, how could a teacher break the expectation that she or he knows what the student is seeing or thinking? I call this a "trick" because it reminds me of card tricks, in which the magician manipulates the attention of the audience. An important role of the teacher is to direct attention appropriately.

In an interview, I told Mrs. Hill about my conversations with Arwa and Tharshini and about the metaphor of the graphing calculators. The following transcript excerpt from our discussion demonstrates Mrs. Hill's awareness of the semiotic tensions I had been discussing with the students:

DW: … how you don't look at the person, but you look at the thing. And then even the thing isn't the thing because it's

Mrs. Hill: a representation.

DW: A representation of the thing.

Transcript 6-8a. Mrs. Hill's pedagogical suggestion.

She was quick to show her familiarity with the vocabulary the students and I had developed in our conversation even though she had been staying out of the conversations for more than a month already.

I described for her the calculator metaphor, and she recognized the problem as being significant in her teaching practice:

DW: That's the way it always is in math, that everyone has their own image of what they see, and we talk about it as though everyone's seeing the same thing.

Mrs. Hill: Yeah.

DW: But they don't.

Mrs. Hill: I know that. I know that, that, that's true. And I know, and it comes out in the tests every once in a while. And you're thinking: holy smokes, how did they get off on that tangent. You know? Yeah. But that is true.

Transcript 6-8b. Mrs. Hill's pedagogical suggestion.

Then I asked her to draw upon her teaching experience to answer a question that I had. I told her about the difference it made for my conversations with students when I no longer held a calculator, and asked how this trick could be done for the semiotic mystery we were discussing. At this point, I had no answer in mind. The question for me was an authentic one.

DW: How can this be done, for the thing I was just describing about how all math is that way? How can you, how can you show yourself not looking at your own image but, I don't know how to ask it, but do you understand?

Mrs. Hill Okay, well I've seen us both do it in class. Somebody will say something. And we'll make it true but it's not what they want us to do. You know, they'll say like, I can't even think of something. "It's a line." And then we'll draw a line and it isn't where they want it to be so then they have to be more specific?

DW: Ah yes. Sort of that obstinate pain thing that I did.

Mrs. Hill: I think so. Yeah. I forgot about that. Yeah.

DW: So you do what they say, but not.

Mrs. Hill: It teaches them that they have to be more specific, more exact.

DW: So you, you sort of pretend ignorance.

Mrs. Hill: Yeah. [*laughing after a short pause*]

Transcript 6-8c. Mrs. Hill's pedagogical suggestion.

Her answer reminded me of an exercise I had had students do in the fifth week of the research. I called the task "obstinate pain." With one student outside the room, I asked another student to follow that student's instructions when he returned. The student had to follow the instructions to the letter but try to avoid doing the thing the other person expects. When the student from outside returned, I asked him to explain to the informed student how to draw a particular figure. The students enjoyed the task. My goal in introducing this task was to draw their attention to careful, non-vague explanations.[12]

Mrs. Hill's suggestion was that teachers be this kind of obstinate pain by feigning ignorance. I agreed, though I doubt, and I expect Mrs. Hill would doubt, that students would be duped by the trick. When I did this in my teaching, students appeared to know that I was playing a game to draw out more careful communication, but they nevertheless played along with the game. This response is not unlike their general complicity in other tasks I would give them. They need not agree that the task is helpful; they may know that I have answers in mind to the questions I ask them, but they play along. They do what a student is expected to do. With the calculator, it was a little different. Students could not see me seeing the graphs that they saw on their calculator screens. And without having the graphing skills that I had, they would not be able to know what aspects of the graph I could visualize.

Mgombelo (2003) has asked a similar question to the one Mrs. Hill and I were addressing: What happens if we no longer take the students to be ignorant, but take them to be *knowers* in the pedagogical relationship? She pushes further and suggests that teachers would do well to take themselves to be ignorant – ignorant of what the students know. For her, this is a way of describing a pedagogical relationship in which the teacher takes the constructivist paradigm seriously. She calls this teacher stance "pedagogical ignorance," and relates it to Mary Boole's description of the core of algebra: it is the acknowledgment of our ignorance, our action in the context of unknowns (see Mason and Spence, 1999).

I do not yet have a satisfying answer to the question Mgombelo and I have asked, but I think it is at the core of quality communication about mathematics. I am thinking about my decision to not hold a calculator when I teach, and my observation that students turned their faces to look at me when I looked to them for descriptions of what they saw in their graphs. This recollection brings me back to the place I started, my surprised reaction to students turning their heads in the videotape. My initial thought when I saw the turning heads was that we need more of this. A turned head suggests that there is something interesting to look at. Students need to be looking from one person's face to another, and from one person's symbolic representation to another person's symbolic representation, in order to see the mathematics through the symbols. I continue to think about how best to do this.

I am not suggesting that students and others should no longer look at their symbols as they face the mathematics about which they are talking. Rather, I see that there are other places to look. I think of my argument with Shauna, for example. She and I used three different diagrams to consider a proof. We turned our faces from one diagram to the other, and we also turned our faces to each other as we negotiated which diagram we should foreground.

Noticing Windows

As we become aware that we are looking through symbols to face mathematics, I think that we make possible a better understanding both of the symbols and of the people around us, for it is in them that mathematics resides.

When I stand by my kitchen window, I rarely look at the glass and the boundaries of the opening; I look through the opening, through the glass. But at times it is useful to look at the glass and at the boundaries of the opening to become aware of the skewed and limited view of the backyard they afford me. In the same way, there can be value in attending to language practice in mathematics class, to understand the problems associated with seeing other people's meaning when we have access only to their symbols. In the context of multilingual mathematics classrooms, Adler (2001) invokes the image of transparency when she refers to the decision about when to attend to language and when to attend to mathematics. She calls the *teacher*'s on-going need to make this decision the *dilemma of transparency. Students* can also experience this dilemma.

In the context of a mathematics class prompted to become critically aware of their language practice, Arwa and Tharshini show that high school students can become aware of the dilemma of transparency. Their observations of mathematics classroom interactions and of parallel interactions at home and in other school subjects testify to the depth of their understanding of the relationship between mathematics and the symbols used to represent it. These two girls turned their faces to look *at* some paralinguistic features of their communication – the nature of their attention and their eye movement – and became aware of the way they looked *through* their mathematical symbols to see the mathematics itself.

The way in which people direct their gaze in mathematical communication, by looking at symbols and thus avoiding human faces, seems to be an expression of the discipline's general disposition to ignore the role of humans in its construction.[13] This depersonalization, in tandem with the linguistic depersonalization detailed in the previous chapter, presents high school teachers with a significant challenge as they interact daily with emerging adults, who are looking for their place in their world. The next chapter details yet another language form that represents conventionality at the expense of personality.

Chapter 7 – De-emphasis in Mathematics Conversation

And you just change it to two square root five.

Some time after a student said this, I asked her classmates, "What does that mean when she says 'just'?" This question began a stream of conversation in which some students latched on to one interpretation and ignored other plausible interpretations that I offered for their consideration. We found that the word *just* is often used in mathematics class, and we realized that while its meaning is unambiguous, it can have varying effects.

In mathematics communication, the adverb *just* can suggest that the procedure or process it describes is simple. It might also be used to direct attention because it suggests that the procedure or process it describes is unimportant, and that another process is more important in comparison. Furthermore, when attention is diverted away from a procedure, it is often because particularities are thought to be commonly understood. A generative tension developed between the students and me as we discussed the effects of the word *just* in the context of communication about mathematics.

Within this research, I have come to realize that the times when I felt most resisted in conversation with the participant students were frequently the most generative times, both for me and for them. However, unsurprisingly perhaps, I did not always appreciate the significance of the students' resistance while I was experiencing it. It was frustrating. The events described in this chapter form one such situation. Like the conversation about agency described in Chapter 5, this one exemplifies the generative nature of active resistance in critical language conversations. After giving an account of my dispute with students about our use of the word *just*, I will discuss the significance of the word in terms of mathematics classroom discourse. I also consider some verbs that can have a similar effect – *go* and *do.*

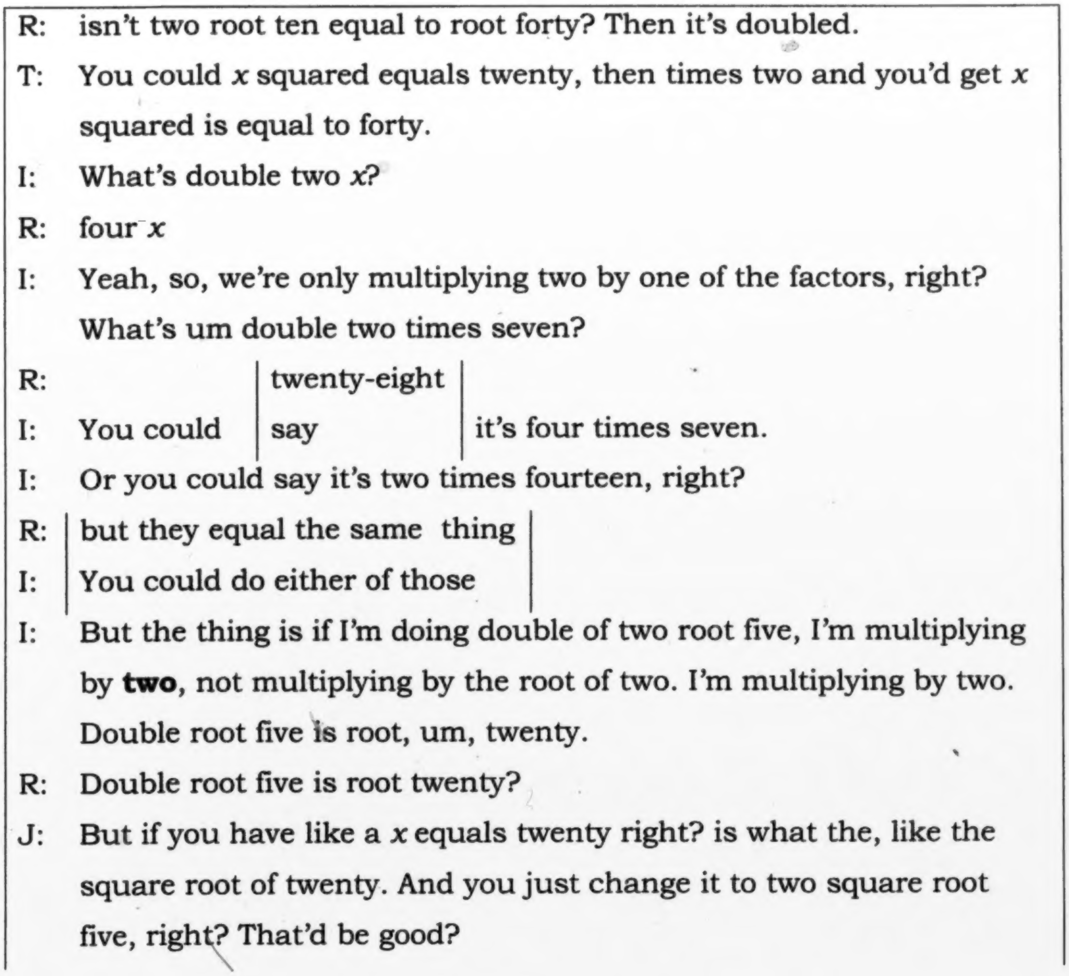
The Story – "Just Go"

Like the story in Chapter 6, the stream of conversation detailed in this chapter had its beginning with my own observations of language features in the researched classroom. I became fascinated with two verbs – *go* and *do*. However, I did not immediately discuss these verbs with the students. The beginning of our interaction about them did not seem to be prompted by anything significant. After giving an account of this conversational stream, I will describe my investigation of *go* and *do* in relation to students' comments.

Noticing *Just*

In the thirteenth week of the research, our discussions about language centred on a problem identified by students. They had said that they did not think with words when they engaged in doing mathematics. In response to this claim, I asked them to write anecdotes that described times when they could not put words to their mathematical ideas. They struggled with this task, so I wrote an anecdote of my own and we talked about mine (which appears in Appendix B). Apparently, their concern for getting it *right* had been a major source of their trouble in writing these anecdotes. It seemed that they were worried about misrepresenting the situations they wanted to describe. Following van Manen's (1997) protocol for researching lived experiences, I said that there can be truth in an anecdote even if we do not quote people *verbatim*. What matters is the person's experience of the phenomenon. Fiction can describe a truth as well as or better than accounts of empirical experiences. On the Monday of the next week, to illustrate the difference between the anecdote and the transcript, I again showed students my anecdote, along with a transcript of the situation I had described in the anecdote.

Figure 7-1 is a copy of the transcript, which I photocopied for them and presented using the overhead projector. "I" refers to me, "T" to Tharshini, "R" to Raina, "J" to Jessye and "M" to Matthew, but Tharshini subsequently told me in our discussion that the utterances I had attributed to her were not hers. In the transcripted conversation, I was trying to answer a students' assertion that $\sqrt{40} = 2\sqrt{20}$.



R: isn't two root ten equal to root forty? Then it's doubled.

T: You could *x* squared equals twenty, then times two and you'd get *x* squared is equal to forty.

I: What's double two *x*?

R: four *x*

I: Yeah, so, we're only multiplying two by one of the factors, right? What's um double two times seven?

R: twenty-eight

I: You could say it's four times seven.

I: Or you could say it's two times fourteen, right?

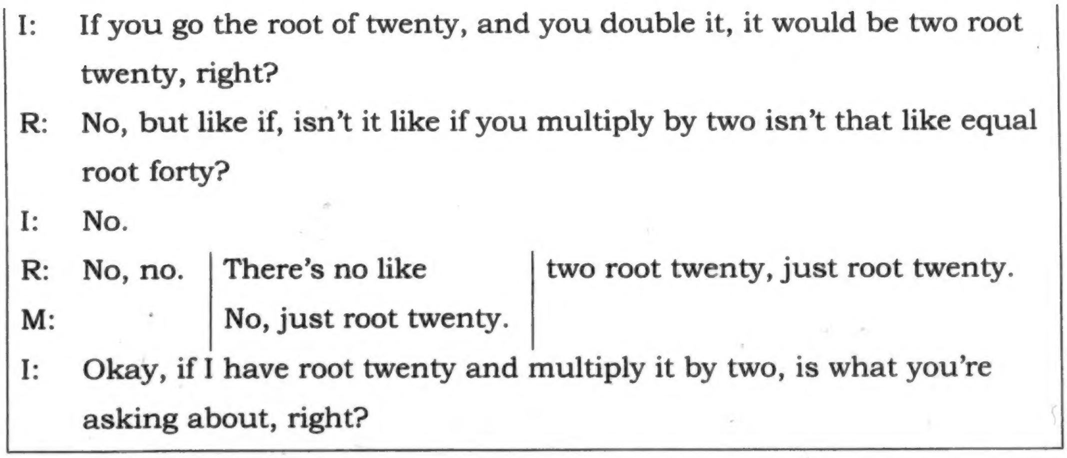
R: but they equal the same thing

I: You could do either of those

I: But the thing is if I'm doing double of two root five, I'm multiplying by **two**, not multiplying by the root of two. I'm multiplying by two. Double root five is root, um, twenty.

R: Double root five is root twenty?

J: But if you have like a *x* equals twenty right? is what the, like the square root of twenty. And you just change it to two square root five, right? That'd be good?



I: If you go the root of twenty, and you double it, it would be two root twenty, right?

R: No, but like if, isn't it like if you multiply by two isn't that like equal root forty?

I: No.

R: No, no. | There's no like | two root twenty, just root twenty.

M: | No, just root twenty. |

I: Okay, if I have root twenty and multiply it by two, is what you're asking about, right?

Figure 7-1. Transcript shown to students in week 14

I asked students if they noticed anything interesting in the transcript. The following transcript details the students' response to my question.

DW: What do you notice about that, maybe in comparison to what I wrote before or from what you remember or anything else? [*long pause*] Anyone find anything particularly interesting? [*pause*] Too bad Jessye's not here. Maybe it's a good thing. No. Um. I just, one thing that I found kind of interesting is this word here – "just." [*circling the word in the transcript, Figure 7-1*] "And you just change it to two square root five." What does that mean when she says "just"? [*long pause*]

Gary: Simply.

DW: Pardon me?

Gary: You simply change it.

DW: Oh. So in other words it's a simple thing to do.

Gary: Well, I guess. I don't know. Well, I guess that's what it's implying.

DW: Yeah.

Gary: You just multiply it by that thing.

DW: It makes it sound easy. Yeah, I was just wondering. I found this interesting because we teachers sometimes say the word *just*. Do you think it kind of is insulting to students? When you say "well you just do this, and"?

misc: [*many students saying things all at once*]

Gary: Well, because you guys have done it for so long. You guys, like, it's not really like, I don't know. It's not that big a deal, but it's kind of implied that we, like, should get it right away, that it should make sense automatically. [*turning slightly to acknowledge Joey's long and clear thumbs up.*]

DW: You're agreeing, Joey? [*Joey nods yes*] Like, Jessye is, like, a student. So it's different coming from her, I think.

Kyle: Yeah.

DW: She's just sort of saying, I guess, that we all know this. This is a simple thing.

Joey: Unless she's hyper-intelligent.

DW: Oh, it's, she's trying to put herself better than you?

Joey: Yeah, it's like using big words.

DW: What if your friend says it to you if you're talking about a math thing between yourselves? [*pause*] Like, would it bother you at all? You'd probably not even notice, eh?

Joey: It'd bother me.

[*silence follows*]

Transcript 7-1. Noticing the word *just*

As I noted in the conversation, Jessye was not present when we talked about her intentions. Her absence was fortuitous, considering my goal for critical language awareness – to uncover a range of possibilities. In her absence, we could hypothesize a range of intentions behind her utterance. If she had been present and able to say what she meant by "just," Gary would have been less likely to offer an explanation.

Though I demonstrated an interest in the word *just* by circling it in this discussion, I cannot recall how I became interested in the word. There is no record of it in my field notes from before the conversation given above. In this conversation, I seem to have been trying to provoke differences of opinion. In my revoicing, I deliberately stretched my interpretation of Gary's intentions, saying "Oh, so in other words it's a simple thing to do," and of Joey's intentions, with "Oh, … she's trying to put herself better than you." This manner of interpreting both verified (or refuted) my interpretation and provoked further discussion.

O'Connor and Michaels (1996) show how teacher revoicing can prompt a participant framework in which students converse with each other. They say that this outcome is especially promoted by revoicing that pits students' ideas against each other. I add that a similar effect occurs when a student voice is positioned against a conventional idea – the authority of the teacher, for example. In this case, my initial revoicing cast Gary in the role of a judge speaking against Jessye's intentions. The second revoicing cast him as a judge speaking against the teacher's warrant to orchestrate and control classroom relationships.

There was a further effect of the revoicing that was significant in this research context, but was not a concern for O'Connor and Michaels (1996), who write about *teacher* revoicing. My revoicing would have changed the way students thought. When Gary said *just* was synonymous with *simply,* he did not say that Jessye's utterance implied simplicity. *I* said that. However, when I asked him if he recognized the implication, he seemed to agree that the implication was present in his interpretation, though he was hesitant at first – "Well, I guess."

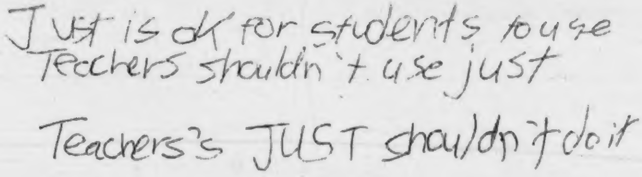
I had told the students that one of my intentions in this research was to listen to the voices of students. I also claimed this intention in Chapter 1 of this thesis. How could I claim to be listening to student voices when I was putting words into their mouths? Though this is a significant question, another question can be used to argue against it:

how can a question be asked or a response be prompted without words that might seed the response? The answer to both these questions is the same – it cannot be done. It is necessary to be careful about the extent to which participants are given words to speak their ideas. And it is necessary to be aware of the influence the wording of a prompt has on the participants' understanding.

As described in Chapter 4, I was finding that the students in this research tended either to say nothing or to make such brief utterances that they could be interpreted in many ways. If I wanted them actually to say something, at times it seemed that they needed to be provoked into speaking. Once provoked, they sometimes began to speak more freely.

It did not take long for Gary to become the spokesperson for the interpretation I seem to have put into his mouth. Joey expressed his agreement with a thumbs-up gesture and defended this interpretation of Jessye's meaning in response to my counter-interpretation. Besides Gary and Joey, a few others students were involved in the discussion, reacting strongly to my interpretation of one of Gary's utterances. Their immediate, simultaneous responses to "Do you find it insulting?" were indecipherable to me both in the audio record and in the moment.

Gary's interest in the word *just* was not short-lived. Two weeks later, when I asked if any student had written anything about language in his or her notes, he remembered exactly what he had written about *just* and exactly where to find it. I photographed this note that he wrote to himself (Figure 7-2):



Just is ok for students to use
Teechers shouldn't use just

Teachers's JUST shouldn't do it

Figure 7-2. Gary's note to himself about the use of the word *just*

This was the first time I had seen the note, yet I sensed that he, Joey and probably others in the class were holding fast to the idea that *just* is permissible for students to use, but that teachers should not use it. I was disturbed by their tenacity even though it was I who had nudged them toward this valuation. With my nudging, I had intended to provoke them into exploring alternative possibilities for using the word *just*, but they did not seem interested in alternative intentions or effects. These students seemed intent on reading malice into this word use.

In Gary's note to himself, there is evidence that he was aware of other uses of the word *just*. I sense him revelling in his own wit as he wrote, "Teachers JUST shouldn't do it." Our conversation was about the word *just* when it is used in a particular way, to suggest that there is no hidden complexity. The *Collins Cobuild Dictionary* (2001) describes twenty-five different contemporary meanings associated with *just* when it is positioned as an adverb. The meaning we discussed in class seems to be described best by this dictionary as follows: "You use **just** to indicate that something is no more important, interesting, or difficult, for example, than you say it is, especially when you want to correct a wrong idea that someone may get or has already got" (p. 845).

A Response to Student Concerns

Like Gary, I wrote some notes on the word *just* after our conversation about the word. Gary seemed to have a pedagogical concern as he worried about the sense of inferiority teachers might engender by using the word. Though I felt responsible for this vein of concern, which occurred partly because of my heavy-handed verbal prompting, I resisted this complaint. I thought it was wrong to attribute only one effect to this use of *just*, and I also felt a little defensive, for Mrs. Hill and I were using the word regularly when we taught.

In order to convince the students of another more positive perspective on its use, I wrote a 600-word essay for them, referring to *just* and *simply* as "diminutives," because they suggest that the actions they describe are unimportant or trivial.[14] I gave a copy of this essay to each student on the day after the conversation about *just*. I told them that it was a concise and tentative draft of a paper I thought I could publish. In Appendix B, there is a copy of the essay as I gave it to the students. It is called "'Just go': Diminuendo in mathematics classroom discourse."

The diminutives *just* and *simply* can be used for pointing, I said in the essay. The de-emphasis of one procedure can emphasize another procedure or another aspect of the reasoning. With such emphasis and de-emphasis, we point attention to the important ideas we are talking about.

In the essay, I extended this idea to include the verbs *go* and *do*, because they, like *just*, can be used to gloss over the complexity of the mathematics being represented. The following classroom situation illustrates this use of the verb *go*. Mrs. Hill was solving the equation $3x^2 + x - 2 = 0$ by factoring. When she said, "[W]e go three x minus two, x plus one," she seems to have used the verb *go* to gloss over the way in which she found this equivalence. As with the suggestive order in which I placed the dots to form perfect squares (see Figure 6-4), the order in which she wrote the expression before saying it would have been suggestive to a student who knew what she was doing, or, in other words, where she was *going.* Mrs. Hill wrote the empty brackets first, then placed the leading terms in each bracket, followed by the constant terms and finally the plus and minus signs between each pair of terms. A student who was not predisposed to know the significance of this order would not have recognized the procedures she was employing.

She could have said the following instead:

> We recognize the left side of the equation as a potentially factorable quadratic so we place two sets of empty brackets side-by-side. We write two factors of the x-squared term inside the brackets. They are three x and x. We multiply the x-squared coefficient and the constant to get negative six. We find a pair of integers with product negative six and sum one, which is the x-coefficient. This pair is three and negative two. We ask what is multiplied by the three in three x to get one of the key numbers, three and negative two. The answer is one, so we place that positive one in the opposite bracket ….

A lot was going on when Mrs. Hill simply *went* $(3x-2)(x+1)$. Even in my long-winded beginning of a replacement for her verb *go,* more could be said. To illustrate, someone might well ask how to find the pair of integers when it is said that "we find a pair of integers."

In addition to the mathematical complexity masked by the verb *go* in the above example, complex *relationships* were masked by the personal pronoun *we.* To me it feels more natural to say to a class "we go three x minus two, x plus one" than to say "we recognize the left side of the equation as …." Why the discrepancy? There can be no definitive answer to such a question; one can only suggest possibilities. When Mrs. Hill used the verb *go* to replace a complex set of actions, she expected the students' complicity, their understanding of the unsaid things that were being done. With this complicity in mind, it seems reasonable to say *we* because it suggests that we are part of a community of people who do not need to say certain things. By contrast, if Mrs. Hill were to have carefully spelled out her thoughts and actions, she would not be requiring the same degree of complicity.

The verb *go* might be seen as a generic verb, because it fills in for much detail. The following example illustrates how the verb *do* can be used similarly. When Mrs. Hill was writing the zeros of a polynomial function expressed in factored form, she said "I'm doing them in order here." She could have said, "I'm reading the zeros from each factor, and writing them in order from least to greatest." The verb *do* replaced the unsaid "read" and "write" as well as the unarticulated and undemonstrated procedure she used to read a zero from a factor: setting the factor equal to zero and solving for x. In this way, *do* can serve as a generic verb, like *go*, replacing another more descriptive verb and the action it implies, or taking the place of a series of verbs and related actions. Such replacements can be used to gloss over complexity, a function that the adverbs *just* and *simply* can also serve. The familiarity of the expressions "Just do it" and "Just go" demonstrate the close relationships between these generic verbs and the simplicity-indicating adverbs.

The night I wrote the essay about the word *just*, I was not able to access adequate scholarship that described the particular use of *just* that the students and I had in mind. I had the *Collins Cobuild Dictionary* (2001) at hand, which I quoted above. This dictionary flags the word as relating to emphasis, but it seemed to me that the word does the opposite of emphasize. In my essay, I chose to call it a *de-emphasizer* for this reason.

Though the word *emphasis* might be seen to refer to the direction of attention in general, and though it implies that attention is being directed toward particular objects or ideas and away from others, the word can also refer specifically to the *positive* aspect of directing attention – directing attention *toward* something. In introducing the term *de-emphasis,* I wanted to highlight the inherent relationship between stressing and ignoring.

After writing the essay for the participant students, I looked through linguistics scholarship to find a better way of describing the particular use of *just* that we had in mind. When Channell (1994) differentiates between vagueness and ambiguity, she describes vagueness as a speaker avoiding precision or exactness. With vague language there is no clear meaning associated with the words, whereas with ambiguity there are two or more clear meanings. My use of the word *just*, and the use of the verbs *go* and *do* as generic replacements for clearer verbs, fit Channell's definition of vague language. These language forms enable a speaker to avoid clarity and precision.

In giving examples of vague language, Lakoff (1970) explains that to avoid particular details is part of vagueness – for example, the statement "Harry kicked Sam" is vague because "Harry could have kicked Sam with either his left foot, or right foot, or both; it is left vague which" (p. 357). In the same way, when a speaker replaces more detailed explanation with the generic verbs *go* or *do*, or when a speaker indicates a general lack of detail with the adverb *just*, the lack of detail suggests vagueness.

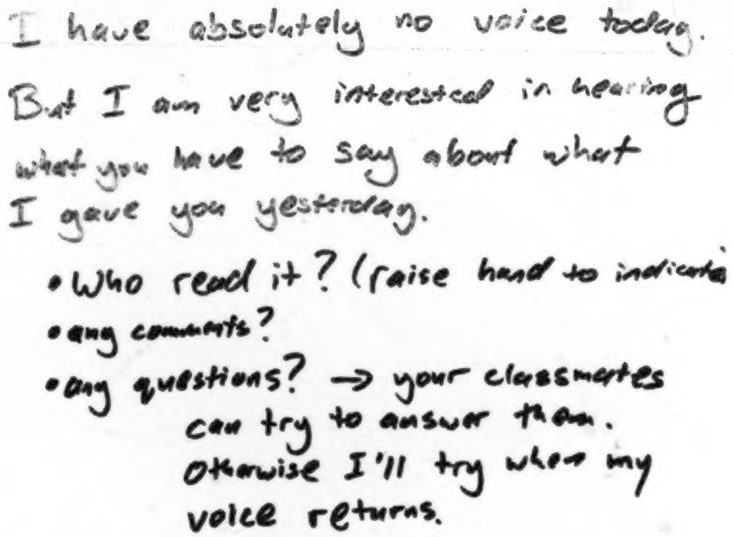
It is more difficult to prove something non-existent than to prove that it exists. Though it cannot be said with complete confidence that no linguistics scholarship considers the use of the adverb *just* that the participant students and I had in mind, I can say that I have not found any discussion of this use in Channell's extensive consideration of language forms that indicate vagueness, nor in more general studies of language practice, including pragmatic analysis (e.g. Levinson, 1983) and semantic analysis (e.g. Lyons, 1977). This use of the word *just* also seems to be left unnoticed in mathematics education scholarship that considers lexico-grammatical forms. The word appears many times in Rowland's (2000) transcripts, with which he exemplifies forms of vagueness in school mathematics communication, yet he does not take note of the vagueness supported by *just, do* and *go*.

Students Respond Again

The essay I wrote for the students ended with two questions:

1. Is there something special about diminutives in mathematics classes? What is it about mathematics that makes this language practice unique?
2. With issues relating to diminutives in mind, what might a mathematics teacher do differently? And, what might a student do differently?

The day after distributing the essay I had intended to discuss it with the students, but I lost my voice. However, I was able to use my enforced silence to prompt students to speak. The students were accustomed by now to a few minutes of discussion about language at the outset of every class, so I used that time to project some questions on the screen using the overhead projector. I revealed the questions one at a time. Figure 7-3 is a copy of the image I projected for them.



I have absolutely no voice today.
But I am very interested in hearing
what you have to say about what
I gave you yesterday.

- Who read it? (raise hand to indicate
- any comments?
- any questions? → your classmates
  can try to answer them.
  Otherwise I'll try when my
  voice returns.

Figure 7-3. Written questions in lieu of my voice

Joey began the discussion by noting that I was replacing my spoken words with written words today. Mrs. Hill tried to speak for me, though she usually stayed out of our conversations about language by this point in the term. The following transcript represents the discussion that ensued.

Joey: Oh, that's what you're trying to say?

Mrs. Hill: Yeah.

Joey: Okay, I get it.

[*Students laugh. Then there is a pause as students read.*]

Joey: Um, yeah. About, about *just*? I thought that the way you put it was very good. I think that it makes, it like makes you relatively inferior to the person who's, who's doing the talking, i.e., but then again, it shouldn't really matter because the teacher's been doing math for a long time so they've obviously earned the right to say like "you just do this" because they know how to do it. You on the other hand, you're like the first time doing it so it's kind of tough doing the things and crap easily. The other person is relatively new to mathematics.

Gary: I disagree [*quietly*]. I disagree [*loudly after being recognized*]. I don't think you should say, like as a teacher, I don't think you should. For, like yesterday with the proofs, Mr. Wagner would say like "just do this." I'd be like, "okay, yeah that's great, because you've been doing it a lot longer than I have," so obviously it's going to be a lot easier for you. Obviously it's going to make a student feel inferior. Joey doesn't think it matters, but I think it does. Like, it's not so important if among, like, peers, like if I were showing Kyle, "Kyle, you can't do this because the angle's not enclosed," because me, Kyle and I, have been doing it roughly the same time, so if I think it's easy, then maybe it should be easy for Kyle too. But even then it shouldn't just be assumed. But especially not if it's a teacher.

Mrs. Hill: Does tone matter? Tone?

Gary: Yeah.

Joey: Yes. [*long pause*]

Transcript 7-2a. Further discussion about the word *just*

Joey seems to have changed his mind in the two days preceding this discussion, because he had earlier claimed that it was insulting for teachers to use *just* to imply simplicity. Though I was hoping that I could convince some students that it would at times be appropriate for teachers to use *just* in this way, I did not like Joey's justification. Was he saying that I, as a teacher, had earned the right to insult my students? I would not want to be seen that way. With my laryngitis, I could not check his meaning as I had two days before. I literally could not revoice his utterance.

Gary disagreed with Joey, returning to his earlier idea, the idea that I seem to have seeded in our discussion. Gary knew that I enjoyed provoking students into disagreement with each other. In an earlier interview he had asked me, "You like it when we argue, don't you?" Yes. Though he was aware that I enjoyed a good dialogue with diverse opinions, it seemed to me that his argument regarding the word *just* was driven by conviction, and not by his desire to give me what I wanted.

The above exchange terminated in silence. It continued when I revealed my next question, "Any questions? Your classmates can try to answer them."

Matt: Maybe um, if there's, like if *just* kind of makes something seem more simple, is there stuff that we use that makes um parts seem more complicated? [*long pause*]

Mrs. Hill: That's actually a good observation. Sometimes you say it to make something more simple – "You just do it this way." And you're showing an easy way.

Joey: I think tone and emphasis on what you're saying, based on, you emphasize certain parts in whatever you're saying showing the students that this problem is difficult, like this is hard or something. And the number of times you explain something. So, if they believe that this should be fairly simple, you'd just say it once, one or two times, and that's it. But if you think that people are probably going to have problems with it they'll say it like over and over again.

Mrs. Hill: Okay? Right [*laughing*]

Gary: That went well.

Transcript 7-2b. Further discussion about the word *just*

Matt's question related closely to part of my essay on *just.* I wrote, "The de-emphasis of equation-solving procedures emphasizes the circle geometry reasoning. With our ways of emphasizing and de-emphasizing, we point attention to the important ideas that we are talking about." Joey's response to Matt focused on another way of emphasizing, with repetition and with tone. It seemed from the tone in his response that he was using the word *emphasize* to refer to inflections of tone and volume: "[Y]ou *emphasize* certain parts."

Gary was pleased with the discussion, summing it up with the evaluation, "That went well." He seems to have been wondering how a class conversation could be conducted in the absence of the teacher's literal voice. Considering his pleasure and participation in this conversational strand, it seems that Gary's concern about students feeling inferior when teachers say *just* was not a concern for himself. He did not show signs of feeling inferior. Indeed, he exuded confidence with his willingness to share his interpretations of language practice and to disagree with Joey.

It appears that Gary was more concerned for other students than he was for himself. Perhaps his empathy for his classmates and students in general arose from a sense of solidarity amongst students, a sense that could be related to the way students perceive schooling. I sensed Gary felt that schooling positioned him and other students against already-determined culture, and particularly against teachers who are the voice of this culture, the voice of the discipline. The expectation that students will surrender human agency to disciplinary agency pervades traditional schooling.

The conversational strand described above came to a temporary close the next day, when I asked students if I was right in saying that they think it best for teachers to avoid using *just*. I still had laryngitis, but the prompting power of my voicelessness seemed to have worn off. Most students chatted with their friends while I used gesticulations to try to maintain their attention. Some students answered the question, but the general din obscured their responses on the audio record. In my field notes I recorded some of the responses.

Of the students I heard, Van spoke first. He laughed and said, “I don’t care.” Apparently he thought his sense of self-esteem would not be affected by his teachers’ word-choices. Signot answered, “I don’t care” and added, “[I]t helps you know when to pay attention.” He seemed to agree with the good reasons for using *just* that I described in my essay.

Resurrecting the Argument

Though the students indicated with their passive resistance that they were no longer interested in the word *just*, Mrs. Hill and I continued to be annoyed by their disapproval of teachers using it. In an interview on the day Gary showed me his note to himself (Figure 7-2), she and I commiserated about the students’ stubbornness, their apparent refusal to accept the validity of the word *just* to imply simplicity. Indeed, we were still using the word every day when we taught, and the students did not seem to notice.

Mrs. Hill: I think they actually came away with the wrong conclusion with *just*.
DW: Because they think it’s bad.
Mrs. Hill: Yes.
DW: I kept saying it again and again, that it’s not bad.
Mrs. Hill: I know.
DW: But
Mrs. Hill: But they reinforced it again today.
DW: Yeah.
Mrs. Hill: That when a teacher says *just* it’s a little bit condescending and whatever.
DW: I agree with you that it’s the wrong conclusion, but on the other hand [*Mrs. Hill starts to laugh*] it’s *their* conclusion, and they’re actually displaying quite, displaying significant resistance to my interpretation.
Mrs. Hill: Hm-hm. [*yes*]

DW: Which may be a significant thing that says that this is an area where their perspective is different from the teacher's perspective.

Mrs. Hill: Yeah.

***Transcript* 7-3**. Mrs. Hill and I notice the students' strong voice

One of the reasons I appreciated my conversations with Mrs. Hill was that she sometimes said things that I was thinking. I found it easier to think critically about these ideas when I heard them in someone else's voice. In this interview, she uttered the complaint that I had been carrying with me for two weeks, that the students had come to the wrong conclusion about the word *just*, or at least that they had not come to realize the full potential of the word. When Mrs. Hill said this, I realized for the first time that the students' resistance was in fact evidence of their independent voice, their human agency in our discussions about language. This voice was one I was purportedly seeking throughout my interactions with them.

I decided to resurrect the argument to see if any students disagreed with Gary. In my second-to-last day with the class, I asked them if they would act out a panel interview, like on televised news programs. I asked two students to play the part of experts, a student and a teacher. I selected the actors because no one volunteered. Arwa played the part of the teacher and Neeta the student. I played the moderator. Gary was absent that day.

DW: We have two experts here coming in, one from Toronto, Ms. Arwa from Toronto, from some kind of school [*laughter*] and a student from Edmonton, Neeta. Um, and the issue is that some experts are saying that there's a problem with teachers using the word *just*. Is that a problem at all, Neeta?

Neeta: No.

DW: What about you, Ms. Arwa.

Arwa: Um, I don't really think I should use *just* as much as a student uses it because I think depending on what we're actually trying to say, you use it and not avoid *just.*

DW: Okay. And Neeta, do you disagree with her, or um you're not one of these students that's

Neeta: I think that when a teacher uses it it's more, like they say "just do something like this," it's kind of, it's kind of chewing them down. I guess. Because if you don't understand it, it kind of makes you feel stupid.

DW: Do you have a response to that Ms. Arwa?

Arwa: Um, well we're trying to clarify that, like "just do that," it's not very clear. Well, you see it that we're trying to point it directly at you guys, but I guess you guys think we're trying to make you feel bad. [*pause*]

DW: And you're agreeing now Neeta?

Neeta: Sure.

Transcript 7-4a. Panel discussion about the word *just*

The interview broke down because Neeta agreed with what we assumed was the teacher's point of view. Because Neeta did not feel capable of feigning criticism, we decided to replace her with Brandon. The interview continued.

DW: Okay. So we have a replacement expert up here: Brandon from Bruderheim. And um, Ms. Arwa is saying that it's okay for teachers to use *just.*

Brandon: I disagree.

DW: Why?

Brandon: Because it's *just* not right. [*laughter*]

DW: By the way, do you really disagree or are you just playing the part?

Brandon: Well it really doesn't matter to me. [*laughter*]

DW: Well, this isn't going so well.

Brandon: Well, I don't get angry if the teacher says *just*.

DW: Does anyone?

Signot: No.

DW: Because when I was out in the hall interviewing you guys most people who said that they noticed it being used in class sometimes thought it was no big deal. So is it only Gary who disagrees?

Many Students: Yeah/Yes.

Tharshini: Strongly.

Transcript 7-4b. Panel discussion about the word *just*

Brandon had the same problem that Neeta had. He claimed that the word *just* did not bother him at all, and he could not adequately play the role of a critical student. When I turned to the class to ask if it was only Gary who found the word problematic, the students replied with a chorus of yeses. Tharshini emphasized the point with her addition, "Strongly." Gary strongly disapproved of teachers using *just*, and his classmates were well aware of his stance.

I felt relieved at this point, thinking that the majority of students agreed with Mrs. Hill and me, but I wanted further confirmation. As described in Chapter 3, I had a sense that the resistances I felt in my conversations with students would point to significant findings in the research. In this chapter, as in Chapter 5, the disagreement between the students and me was about certain language forms in our mathematics conversation. In this case, Mrs. Hill and I were frustrated by the students' apparent determination to focus on student insecurities that were highlighted when teachers used the word *just*. Though I recognized the value of the students' resistance to my interpretations, I admit to a feeling of achievement when I felt that I had won them over. Ironically, I now find that success in the research came from the events in which I felt the least successful as a teacher –

events such as this stream of conversation in which I was unable to convince them of a benevolent use of the word *just*.

I asked again if anyone agreed with Gary. In response, Jocelyn added yet another interpretation. Her interpretation highlighted another complaint students might make about their mathematics teachers.

DW: Anyone else?
Jocelyn: Well, I kind of think that when they use *just* it's kind of an aggressive word. It's kind of like they just use *just* because they don't want to explain why it is, they just say, "It's just that."
Tharshini: Or maybe they don't | have time. |
Jocelyn: | And they don't | want to explain other ways.
Tharshini: Or maybe they don't have time | to explain. |
Jocelyn: | They say, | "Just do it this way."
Signot: I don't really care what they do.
Jocelyn: It's kind of saying that there's only one way to do it.
Kyle: Yeah. That makes sense.
DW: Or there's one way that we're going to do.
Tharshini: Well, yeah.
Jocelyn: But not really giving you the freedom to do what you want.

Transcript 7-4c. Panel discussion about the word *just*

Jocelyn used strong words to describe her contempt for teachers not giving full explanations of the mathematics they demonstrate. She described the practice as "aggressive" and she interrupted Tharshini's responses twice. In light of this evidence of her strong commitment to her interpretation, the apparent low modality of her statement "It's kind of saying that there's only one way to do it" seems out of place. Perhaps her hedging words – "kind of" – are a result of her difficulty in finding words for her interpretation rather than of shallow commitment to it. Indeed, it is unlikely that Jocelyn would previously have heard or participated in conversation about the forms teachers use to avoid explaining. With this lack of experience, it would be unlikely that she would have ready words to describe what she felt.

Jocelyn did not seem so worried about students feeling insulted; she had a different concern. Her use of the adjective *aggressive* might suggest resonance with Gary's concern, but the rest of her argument focused on the way *just* can be used in the place of a more detailed description.

Her argument reminds me of the stereotypical conversation between a two-year-old and a parent. The child asks *why* questions – question after question. The parent tries to give good answers, but finally terminates the exchange with an "it just is" answer. Jocelyn did not appreciate teachers saying, "It's just that." In mathematics classrooms, this scene is commonplace. For example, a student asks the teacher why the product of two negatives is positive. "It just is."

Tharshini countered Jocelyn's legitimate concern, pointing out time constraints. The teacher could not possibly explain everything. Then Jocelyn added that this kind of avoidance of proper explanation suggests that there is only one way of doing the procedure. It constricts the student's freedom. Kyle and Tharshini agreed. I wondered how Jocelyn saw the relationship between avoiding explanations and one-right-path thinking. I did not get a chance to ask her. Kyle and Tharshini readily agreed with Jocelyn's sense of connection between one-right-path thinking and the encapsulating, simplicity-implying *just*, but they may have had different reasons for connecting these two ideas together. I too had come to the same conclusion weeks before talking with the students about *just.*

Jocelyn's concern relates to Grice's list of principles that he notices as being over-arching assumptions that guide people's conduct in conversation.[15] In particular, her concern is an example of the discursive authority of his maxim of Quantity, which states that in normal conversation people follow these rules: "[M]ake your contribution as informative as is required for the current purposes of the exchange" and "do not make your contribution more informative than is required" (Levinson, 1983, p. 101).

It appears that Jocelyn had been disgruntled with teachers who have not, in her opinion, made their oral contributions as informative as required for her purposes. Though her concern is justifiable, her classmates may have "required" a lesser quantity of explanation. When a teacher addresses a class of thirty, it is unlikely that all the students have the same requirements for explanation. Tharshini's responses to Jocelyn's concern correspond to the second part of the Gricean maxim – the teacher should not explain more than necessary. The disagreement between Jocelyn and Tharshini illustrates how the Gricean principles are most evident when they are flaunted.

The above account of my conversation with participant students about the word *just* raises only issues that came up in the discussion. There are other aspects of the adverb that we did not address. For example, when a teacher uses the word in the way we were discussing, there seems to be an evaluative component to it. I alluded to this in my short essay on *just,* but the students did not take up the issue. Teachers can indicate the level of difficulty of a particular mathematical procedure by their word choice. "Then you just solve this equation" suggests that the procedure should be very easy for students.

Alternatively, "Now we have to think about how we can solve this equation" suggests that the procedure will be a challenge for students.

There is also an evaluative component when *students* use the word *just.* They can indicate their familiarity with a particular process or procedure. When a teacher asks a student how she arrived at a particular answer, she might respond, "I just solved for *x*." Such an answer demonstrates the student's evaluation of the mathematics being referred to.

Diminuendo and Mathematics

In the essay I wrote for the students, I equated two ways of drawing attention. We can direct attention to the significant item. Alternatively, we can direct attention away from insignificant items. I called this alternative "diminuendo" and asked two questions about the practice of diminuendo in the communication of mathematics. I am not yet satisfied with my answers to these questions. Indeed, answers to such questions cannot be conclusive. With the first question, I wondered about the connection between mathematics and what I called diminuendo. The second one questioned the possibilities afforded to students and teachers who are aware of diminuendo.

I felt that I had only one day to write the essay for the students, because I did not want them to lose interest in the discussion we had begun. Thus I did not have much time to consider the merits of the word *diminuendo*. I recall choosing it because it is a nominalization, a noun that describes a human action, a process in music-making. I wanted to focus attention on the choices faced by a person communicating mathematics. *De-emphasis* and *emphasis* have the same property, but they did not satisfy me because they seemed so one-sided.

In music, diminuendo refers to a change in volume, which can be a way of changing emphasis. The *Grove's Dictionary of Music and Musicians* (Blom, 1954) defines *diminuendo* as "[a] direction to lessen the intensity of tone in a musical passage, phrase or note. The sense of the word is precisely the same as that of *decrescendo*, but, being positive instead of negative, it is preferable" (p. 708). Some notes become more prominent because they are louder or softer than the others that surround them. Similarly, in mathematics the word *just* can be used to direct attention away from certain procedures in order to focus on other procedures. It is the juxtaposition that is important, stressing some aspects and ignoring others. This juxtaposition makes for an indirect stressing; by diminishing the prominence of the surroundings we can increase attention on the parts not diminished.

Gattegno (1984) asserts that every circumstance of life involves stressing and ignoring. He adds that stressing and ignoring are especially important in mathematics education because they constitute the very process of abstraction. In mathematics, the ignoring is layered with each level of abstraction: "[I]t is possible to constitute a cascade (or hierarchy) of abstractions by stressing attributes or properties and ignoring others in already-stressed items" (p. 34).

For example, when Mrs. Hill glossed over the factorization of $3x^2 + x - 2$ and said "we go three x minus two, x plus one," she used one form of the English language to draw attention *away* from the process by which she did the factoring in order to draw attention *to* ways of solving quadratic equations. She was stressing equation solving, and ignoring the mechanics of factoring. If she were to have stressed the mechanics of factoring, she might have used an algebraic algorithm or a procedure that uses rectangular tiles as icons that represent the terms in the polynomial. Stressing aspects of either of these approaches to factoring would likely involve ignoring the mechanics and meaning of addition, subtraction and multiplication. The layers of ignoring continue. To stress the meaning of these operations is likely to imply ignoring the principles of counting. And counting itself is a method that requires stressing certain similarities between objects and ignoring certain differences.

This process of stressing and ignoring, which is fundamental to mathematics, speaks to the significance of the word *just* in mathematics discourse, because the word is instrumental in directing attention. I suggest that stressing and ignoring are related to agency as well as to attention. Various words have been used to describe the ignored aspects of a person's mathematics, including the words *fluency* and *transparency*.

For example, in the above example Mrs. Hill directed attention away from the process of factoring in order to direct attention toward solving equations. She expected her students to be fluent in factoring. This fluency would enable them to attend to another aspect of mathematics.

Fluency is like transparency. In the previous chapter, I compared language to a window. When we look through windows to see things on the other side, we do not see the window, but it is nevertheless possible to look at the window. When we use language to talk about mathematics, we do not notice our language choices, but it is *possible* to attend to language choices. Similarly, when we factor polynomials to solve polynomial equations we may not notice how we are factoring. The process is transparent, though it need not be. There is value in having this kind of fluency, to be able to use tools such as language and mathematical algorithms transparently.

Jocelyn, with her complaint about insufficient explanation, appeared to be resisting the modelling or promotion of fluent practice. In fluent process, which might be described as thoughtless, she noticed what she called a loss of freedom – "not really giving you the freedom to do what you want." With fluency we *just do* or *just go,* and we forego the possibility to consider alternatives.

Considering alternatives is a part of human agency. When we *just go* along a typical path, we surrender to the agency of the discipline. In order to exercise human agency, we need to attend to processes, procedures or choices that we normally perform unaware. Human agency requires that we look *at* the window as well as *through* it. When a mathematics teacher says, "we just go …," the teacher is the voice of the discipline. For students to develop their human agency in mathematics, they may benefit from awareness of language forms that support disciplinary agency, forms that include words like *just*, *go* and *do*.

Possibilities for Students

Hewitt (1999; 2001a; 2001b), among others, draws inspiration from Gattegno and asks how educators can direct awareness in mathematics classrooms. He reiterates Gattegno's (1987) assertion that "[o]nly awareness is educable" (p. 220) and adds an assertion of his own:

> Educating awareness is not something I can do for someone else. The best I can do as a teacher is to use my awareness of pedagogy, the subject matter and the student to make pedagogic decisions about what I do or do not offer. It is the student who must educate their own awareness. (Hewitt, 2001b, pp. 39-40)

His interest relates to the second question I asked at the end of the essay on diminuendo. What might awareness of linguistic emphasis and de-emphasis and its purposes suggest for mathematics learners and mathematics teachers? My emphasis on the student is appropriate in light of Hewitt's assertion that students are in control of their own awareness. My interest in the teacher is also appropriate, for this person offers prompts and tasks that can open possibilities for students to direct their awareness.

When Jocelyn drew attention to the effects of vagueness in mathematics communication, we had three different but interrelated accounts of the primary effect of the simplicity-implying use of the word *just* in mathematics discourse. Each of these accounts could also apply to the generic verbs *do* and *go*, which imply simplicity or unimportance as they gloss over procedures. First, this use suggests that a procedure is obvious. Second, it directs attention away from the procedure. And third, this diversion of attention glosses over alternatives to the procedure. I feel that Jocelyn, who worried about glossed-over parts of an explanation, came closest in this discussion to meeting my hopes for student critical language awareness – an awareness that opens up alternative ways of living within the discourse.

Gary was interested in the way the word *just* suggests that procedures should be obvious. He saw the teacher's use of the word as a potential source of frustration, but what could he do differently because of this awareness? Perhaps it could mitigate his possible sense of inferiority, although he did not seem to have any sense of inferiority. One could rightly suspect that other students who become aware of linguistic forms in their mathematics classes would share Gary's strong sense of confidence. If such awareness is unnecessary for him, it might even be a distraction.

Signot mentioned briefly that words like *just, go* and *do* help students know what is important and what is unimportant. By this he seems to have meant that the words could help students figure out what the teacher deems important. This awareness of the pointing power associated with emphasis and de-emphasis might help a student or teacher direct attention effectively in communicating mathematics. With the exception of Signot's brief utterance, the students in this study seemed unmoved by the significant possibilities such awareness afforded them.

By contrast, Jocelyn, who was upset by vague language, demonstrated her awareness of a subtext in mathematics communication and opened up new possibilities for herself. She saw that alternative mathematical possibilities were being glossed over, and she could attend to these alternatives even when the speaker might deem them trivial. By being aware of the role *just, go* and *do* can play in masking aspects of the mathematics, she could direct her awareness elsewhere. These three words could cue her attention to the ongoing stressing and ignoring that is at play in any mathematics communication. When she would hear a teacher or classmate say "[J]ust go...," she could say to herself, "Yes, there is an obvious way of doing this, but how might I go about this differently?" This kind of awareness is the goal of critical language awareness – to become conscious of alternative possibilities within the discourse.

Teachers can also benefit from an awareness of diminuendo in classroom practice. Attention to this language effect can make certain kinds of reflection possible. For instance, one might attend to the verbs that the adverb *just* modifies in classroom mathematics. In the transcript that initiated my conversation with students about the word *just* (Figure 7-1), Jessye used the adverb *just* to modify the verb *change* – "you just change it." What else does the adverb precede? That day, in the course of the mathematics lesson, I heard the word *just* four times. Mrs. Hill wanted students to begin a set of exercise questions, and she said, "Let's just do some." This is the typical word pair *just do* that describes the fluency that exercises are supposed to promote. As they worked on the exercise questions, Joey said to a classmate, "You just punch the numbers into the calculator." Indeed, calculators support transparency by allowing us to perform extensive algorithms in an instant, allowing us to concentrate on other aspects of the mathematics. After the exercise work, Mrs. Hill asked Gary to share with the class his solution to a particular question. She asked him to "just tell us in words." She told me afterwards that she wanted to restrict him from writing so that he could concentrate on speaking. When Gary gave his solution, he concluded by saying, "And then x is just 62" – it *just is*. This is the very expression that Jocelyn said she despised hearing – "It just is."

The verbs that precede the word *just* tell us what processes we are asking our audiences to ignore. It is important for educators to be aware of what they are stressing and what they are ignoring.

More than Just Following

In any discourse, it is natural to *just* fit in, to follow the language and behaviour patterns of the people around us. In mathematics class, it is understandable that students would think, "This is *just* how it is done." Alternative mathematical possibilities can become accessible when students come to realize that certain language patterns can actually mask these alternatives. As students develop their sense that conventional mathematics need not be taken as though it is pre-existent, a space opens up for them to improvise and create within mathematics. In this space, they can develop their voices and express their human agency.

In the next chapter, I will consider more generally the way in which human agency appears to be masked in mathematics class discourse. What can an educator do in a discursive setting that typically silences the human voice?

Chapter 8 – Responding to Silence

Why are you looking at nothing?
I do not deal with nothing! I can't name it. It's of no use.
(Edwards, 1999, p. 118)

In the first seven chapters of this thesis, I focused on critical awareness of language as a framework for investigating a range of possibilities for students participating in classroom mathematics discourses. The focus on possibilities for students is significant, considering the many children in this world who study mathematics. This chapter directs attention elsewhere. It is devoted to pedagogical questions and it reflects on the previous four chapters' accounts of the on-going conversation in the researched classroom.

In this chapter, I ask what possibilities might be available to mathematics teachers who are aware of the language issues raised in these chapters. This question reflects my orientation to critical inquiry. While the previous four chapters focus on *description* of what happened and on my interpretation of what happened, this chapter focuses on *examining possibilities*. Although my exploration of what is possible relies on my intuition, it is informed by what happened in the researched classroom. For this reason, when I say in this chapter that something can be done, I am saying that I think it *possible*. In Chapter 6, there is also some consideration of possibilities for teachers, but that is not the primary focus of the chapter. I included the shift of attention from student possibilities to teacher possibilities in the chapter because the diversion was part of the stream of conversation I was reporting.

To turn my attention to pedagogical possibilities is natural for me because I am an educator and a former school mathematics teacher. This focus of attention is most appropriate for scholarship because school mathematics students do not have ready access to the literature. Rather, it is their teachers who may access scholarship as part of their preparation to become teachers and as part of their professional development. Much of these teachers' access to scholarship is mediated through teacher educators like me while some teachers read some of the literature on their own.

In addition to this chapter's shift to exploring pedagogical possibilities, it differs from the earlier chapters in other ways. With its departure from giving an account of the events in the researched classroom, there is a more interrogative tone. I ask questions, and leave some of them unanswered. While some of these unanswered questions may lead to further formal research, my intention here is to present them as prompts for further reflection – for myself, mathematics teachers, educators of teachers and researchers of mathematics learning.

Working With Silence in the Mathematics Classroom

The theme that emerged from this research is the silenced person in mathematics classroom discourse. The students showed their interest in this theme, as each of the streams of conversation that captivated them seemed to have something to do with silence. However, I need to be cautious in my interpretation. While it was clear that some students were very interested in these themes, some of their interest can be attributed to their complicity in my own interest in silence. Just as in the researched classroom we paid attention to language practices that relate to silencing humans, my subsequent interpretation of the on-going conversation has became attuned to silence. While I could have noticed other themes in my interpretations of the particular episodes reported on here, what I noticed were the aspects of silence. Despite these concerns, the conversation in the researched classroom has shown that silence is an important aspect of classroom discourse.

Silences in this Research

Where did my interest in silence begin? As I tried to understand the participant students' overall response to my language awareness prompts, both during and after my interactions with them, their silence spoke powerfully to me. While my awareness of silence appears to have its roots in this research, I have noticed in reflection that silence had been an important theme for me before this research began, though I did not realize it. For example, six years ago I was moved by Krog's (1998) account of South Africa's Truth and Reconciliation Commission because of the role the book and the commission played in giving voice to people who had been silenced by the Apartheid regime, but I am quite sure that at the time I did not think of the book and its context in terms of silence. My conversations with the students in the researched classroom have been the experiential threshold that has made me aware of silence and afforded me a new possibility in the discourses that I am part of. I am now better equipped to speak and write about silence.

As I reiterate the first seven chapters' connection to silence, I will consider possibilities for educators who want to respond to silence in the mathematics classroom. The first three chapters do not draw attention to silence in the way that the next four do. These introductory chapters provide a backdrop for the account of the research. In Chapter 1, I account for my interest in mathematics classroom discourse. Chapter 2 reviews literature relating to the study of this discourse. And Chapter 3 describes my methodological orientation and the processes that set up the researched conversation. With reflection, it is possible to see silences in these accounts of the research context. The silences include personal background that was not mentioned because I consider it insignificant to the research, holes in the scholarship (aspects of the discourse not yet investigated) and alternatives not chosen for the method. However, these silences do not point to the silenced person in mathematics. Rather, they represent aspects of silent discourses.

Chapter 4 is an account of my attempts to prompt conversation about language in the researched classroom. While reflecting on student responses to these prompts, silence became tangible to me. Though their silences in response to my efforts to raise their language awareness might not appear to be directly related to the silenced human, they do suggest a surrender to disciplinary agency, which by definition suggests a diminishment of human agency. Students apparently had difficulty conversing about language largely because to do so would have meant breaking out of typical mathematics class discourse patterns. It would have challenged disciplinary agency and embraced creative, human agency instead. When students remained silent – their characteristic response – they were adhering to typical discourse patterns. But they did not always remain silent.

The Silenced Person

Chapters 5 through 7 describe exceptions to the students' silence regarding language in their mathematics discourse. Ironically, their *lack* of silence in the streams of conversation represented in these chapters revealed a clearer conception of some silences endemic to their classroom discourse. It became clear that mathematics discourse characteristically silences the person.

Chapter 5 is an account of my conversations with participant students about voice and agency. The students spoke as advocates of language practices that obscured human agency in mathematics communication. Their reflections on their mathematics in this and previous years demonstrated their sense of surrender to what Pickering (1995) calls the *agency of the discipline*. I found this surrender lamentable and agreed with Boaler (2002; 2003), who advocates engaging students in a dance of agency in school mathematics, a fluency of movement between human and disciplinary agency. This stream of my conversation with participant students exposed a silence in school mathematics discourse – the silenced human voice.

Silences define a discourse as much as its utterances do. The line between the silences and the utterances may be seen as the boundary of the discourse. Because silences are inescapable, they should not be evaluated as always bad or always good. Rather, there is value in becoming aware of the silences and reflecting on possible ways of accepting and responding to them.

The silencing of the human voice is an important part of mathematics. It is one of the silences that make the discourse what it is. By way of illustration, in Balacheff's (1988) description of school mathematics practices of proof, he notes that higher level proofs are marked by the disappearance of the person: "The move into conceptual proofs requires an altered position: the speaker must distance herself from the action and the processes of solution of the problem" (p. 217). He continues describing the nature of the language of proofs, which he says require decontextualisation, detemporalisation and depersonalisation. They involve "detaching the action from the one who acted and of whom it must be independent" (p. 218).

Though depersonalisation is a characteristic of mathematics classroom discourse, it need not be enacted thoughtlessly. I suggest that students can better understand proof if they are aware of the depersonalisation involved in it. I find it odd that depersonalisation is a characteristic of mathematical proof, while proof itself is a highly relational act. Proof requires an audience and a subject; a person proves something to someone else. In response to this odd discursive practice, teachers might point out the depersonalisation that is conventionally associated with proof and ask when it might be appropriate to mask human agency – in mathematics discourse and elsewhere.

On the other hand, as Adler (2001) notes, there is value in leaving aspects of language transparent. Pimm (1987) describes an effect of depersonalisation that would be nullified by attention to language forms that silence the person. He addresses and legitimizes the common fear of involving and exposing the self in mathematics.

> The public image of mathematics is of something objective and absolute, permanent and impersonal. The inner mental activities of an individual are subjective, partial and relative. In the light of such beliefs about the nature of mathematics, it would therefore be reasonable to consider it not only appropriate, but the only proper way of behaving, for a teacher to refrain from exposing any personal images or thoughts. (p. 70)

He seems to be saying that it is because of the nature of the discourse, the characteristic impersonal forms of mathematics, that it is so difficult for some students to expose their personal voices within the discourse. Though it is necessary for teachers to be sensitive to the real fears their students face, I ask how the public face of mathematics discourse might better invite human agency without educators either intentionally modifying language practice to include human voice or explicitly discussing the nature of the depersonalisation within the discourse.

While I am advocating that mathematics teachers become more aware of language practice, I recognize challenges in such awareness-building. It is a dilemma – Adler's (2001) dilemma of transparency. With this dilemma in mind, I suggest that educators can benefit from reflecting on the way they handle it. They can ask themselves: How do I draw my students' attention to their language practice within the discipline and to what end do I do this?

Chapter 6 is an account of my conversations with the participant students and their teacher about the places we look when we communicate mathematics. I suggested that the tendency to look at (or through) the mathematical symbols, which involves looking away from the face of the person, is an embodiment of the silenced voice. The hidden (or ignored) literal face is like the hidden (or ignored) human voice in mathematical utterances. This turning of the face is only one aspect of the embodiment of depersonalisation. We might ask what other embodiments there are? When we surrender to disciplinary agency, to the discipline of mathematics, what do we do with our bodies besides looking away from the person?

Sterenberg (forthcoming), in her phenomenological investigation of the experience of getting the right answer in school mathematics, differentiates between two senses of being right – the Latin *veritas*, which suggests alignment with an impersonal, external ideal, and the Greek *aletheia*, which suggests a more internal sense of fit or beauty. I suggest that this differentiation parallels Pickering's (1995) contrast between disciplinary and human agency. In the context of a classroom culture dominated by the *veritas* sense of rightness, Sterenberg contrasts the bodies of students who get mathematics right with the bodies of those who get it wrong:

> Our bodies become slumped and lowered when we are incorrect. We are unable to straighten our bodies. How different this is from a student who is confident he knows the correct answer. Our experiences of getting the correct answer are connected to notions of straightness and uprightness. The student who is right stands straight because he knows he is correct. It is not an effort for him. Our experiences of correctness are embodied.

The student who is right positions his body upright, erect and at a right angle with the earth. What is the significance of this posture? It seems odd that surrender to disciplinary agency manifests itself in what we often interpret as proud bearing. This phenomenon is reminiscent of soldiers on parade. The ideal soldier has no human agency. He follows orders.[16] He embodies his surrender to authority by standing at attention. Whether it is a soldier or a mathematics student who is erect and attentive, it is appropriate to ask what he is attending to. Is he like a waiter, attentive to the discipline, ready to follow its beckoning? Or is he attending critically, questioning possibility within the discipline, and being creative as only a human can be?

These questions about posture suggest that teachers can benefit from watching their students' bodies. While students turn their faces away from each other to face the mathematics, teachers can turn their own faces toward their students' bodies. While mathematics is the proper object of attention for students in mathematics classrooms, the students themselves are the proper objects of attention for teachers.

Christiansen and Walther (1986), in their investigation of tasks for students, differentiate them from the task of teachers. They assert that the real object of the teacher's professional work ought to be "students in activity with mathematical tasks" (p. 306). Perhaps teachers who attend to the bodies and the utterances of their students will be enabled to evaluate whether they themselves have surrendered to the stereotypical teacher discourse, which Davis (1996) describes as follows: "[W]e have created a system that values compliance over creativity, that spawns destructive behaviour by destroying our experience, and that conditions learners to reach for the formulaic ahead of the imaginative" (p. 281).

There is a sense of martial order when a group of people surrenders to a discipline. If we value human agency, perhaps we need to welcome the apparent disorder that accompanies it. Pickering (1995) speaks about surrender to disciplinary agency, but it may seem odd to speak about surrender to human agency because surrender implies giving oneself up. However, from the perspective of an educator, the word *surrender* seems appropriate. In order to support and encourage human agency in our students, we need to surrender to the students' voices to some extent. Because we represent the voice of the discipline, we surrender to human agency by restraining our own voice. We need to surrender control in order to promote the apparent disorder that is an embodiment of human agency.

Bakhtin (1936/1984), in his revolutionary interpretation of the work of Rabelais, asserts the importance of understanding the carnival in Rabelais' time. I suggest that the antithesis to depersonalisation in mathematics classrooms would share elements of the carnival: "Carnival is not a spectacle seen by the people; they live in it, and everyone participates because its very idea embraces all the people. [...] During carnival time life is subject only to its laws, that is, the laws of its own freedom" (p. 7).

The promotion of such carnivalesque moments in mathematics classrooms does not imply the rejection of disciplinary agency. As Bakhtin says, "[C]arnival is the people's second life, organized on the basis of laughter" (p. 8). There must still be a first life. Carnival was intimately connected with the ordered church liturgy, with the structures of society. It would not have made sense outside the context of structure. Bakhtin was not rejecting the importance of the regular rituals and order of life. Rather, he was pointing out the problems with interpretations that ignore carnivalesque complements to this order.

Lensmire (1994; 1997b) uses Bakhtin's (1936/1984) interpretation of Rabelais' carnival to understand a year of workshop writing in his grade 3 classroom. And Phillips (2002) finds Lensmire's work helpful in her reflections on the classroom experiences that related to her project. When she had her grade 4 students write a mathematics textbook for the next year's class, she noticed that the topsy-turvey aspects of this task were also present in "normal" classroom interaction at times. In this sense, carnival is normal, and may not seem special at all.

However, I suggest that Bakhtin's (1936/1984) notion of carnival does not imply that the carnival is abnormal. Rather, Bakhtin asserts that carnival was normal in Rabelais' world. It was a complement to the more structured aspect of the society. While it is important to *include* such complementary experiences of disorder, it is more important to *recognize* the value of these experiences and to *capitalize* on them. Houssart's (2001) account of a rival discourse in a mathematics classroom – whispering elementary students – seems to exemplify the significance of the carnivalesque. While the teacher doing the research did not acknowledge the significance of this discourse, Houssart demonstrates the potential contribution the discourse could have made if it had been valued.

When stereotypical relationships are turned on their heads, students are afforded significant opportunities for understanding the typical structured aspects of the discipline. Investigations of possibility are such moments. Critical explorations of alternatives to language and mathematical structures can help students understand these structures, improvise within the structures and push at the boundaries of the structures. As Lensmire (1997a) notes, carnivalesque moments are "sites of possibility."

At the end of Chapter 6, I described a need for school mathematics tasks that direct students' faces to each other and to the various symbolic expressions that others use to represent their mathematics. Now I am recognizing the apparent disorder that this kind of mathematics would promote. However, just as Bakhtin recognizes that a society cannot be understood without its carnival, I suggest that mathematics cannot be properly understood without a dance of agency, a fluent movement between human and disciplinary agency. While there is still a need for attention to the discipline, it needs to be tempered with attention to the human.

The thread of conversation reported in Chapter 7 related to the reciprocal relationship between human and disciplinary agency. While some students seemed unable to move beyond particular intentions that could be associated with what I called *diminuendo*, I was trying to show them how diminuendo can be a tool for directing attention away from one aspect in order to focus on something else. As a representative of the discipline, I resisted their voices, which were an expression of human agency. Furthermore, I was defending the need for diminuendo, which is a technique that expresses fluency. And fluency is, in essence, an expression of surrender to disciplinary agency.

While most students in this part of the researched conversation spoke against surrender to disciplinary agency, some of the same students spoke against the expression of human agency in the stream of conversation reported in Chapter 5. Clearly, it is a dilemma for students to negotiate the path between these two apparently contradictory forces. A teacher has an opportunity to help students find their way in this tension. Boaler's (2002, 2003) use of Pickering's (1995) terminology, where she calls classroom life in this tension a *dance*, is a good place to start. The imagery of dance can make the tension feel healthy and generative. In dance, the tension is what makes us feel alive and human.

I suggest that awareness of diminuendo can also help explain the complementary relationship between human and disciplinary agency described above. How can a human exercise agency in the absence of a discipline? Or, using the metaphor of the dance, how can a student dance between human and disciplinary agency if there is no discipline? The human needs a dance partner.

Heinz von Foerster also uses the metaphor of a dance to underscore that conversation is necessary for the realization of humanness:

> It's the humanness which is expressed in the conversation that is so important. […] Because in the reflection, in the eyes of the other, your own humanity begins to develop. Which you cannot do in a

> monologue. You have to dance with somebody else to recognize who you are. (quoted in Waters, 1999, p. 82)

As an example, for a student to investigate the relationships between angles in a cyclic quadrilateral, she requires fluency with addition and subtraction. If the student were to attend to her human agency in performing addition, pausing to consider a range of alternative conceptions of addition, she would not also be able attend to the relationships between the angles. By contrast, if she surrenders to disciplinary agency with regard to addition and employs conventional algorithms fluently, she can exercise her human agency when she considers angle relationships.

The educator's response to becoming aware of the silent human in the mathematics classroom should not be to abandon conventions within the discipline. Rather, a proper response requires a balance between disciplinary and human agency.

Other Silences in the Mathematics Classroom

The above paragraphs consider possibilities for teachers working in this discipline that typically silences the person. In addition to this silence, which is inherent in the discourse and even defining of it, there are other forms of silence in the mathematics classroom.

One of the important silences in the mathematics classroom is gendered. It is often said that girls are literally quieter than boys in mathematics classrooms. Though I have found the reverse to be true in my previous mathematics teaching experiences, the researched classroom described here clearly exemplified the stereotype. I asserted above that silences help us see the boundaries and definition of a discourse. We might become riled at the idea of the silenced female being a part of the definition of mathematics.

However, I assert that it is necessary to investigate the possibility. What evidence is there that mathematics is a masculine discipline? Walkerdine (1988) is one researcher who has found evidence that it is. The passivity and reproduction that she demonstrates to be expected of girls could be described as surrender to disciplinary agency, surrender that she sees as marginalized in current scholarship relating to mathematics teaching practices (Walkerdine, 1998).

In response to an awareness of the gendered definition of mathematics, educators may consider alternative ways of living within the discourse and thus change the discourse locally and, eventually, globally. For example, Pimm (1993) has thought about alternatives to proof and rigour after noticing that "[w]ords and phrases such as rigour and rigid, mathematics is hard, the power of mathematics, 'Mathematics is a stiff discipline,' can be heard as arising from a male sexuality" (p. 37). To consider alternative possibilities in this way is the essence of critical awareness.

Having listed some of the silences I have noticed in mathematics classroom discourse, I recognize the difficulty in sensing them and describing them. It is easier to describe something that is there than it is to describe something that is absent. For example, as I said in Chapter 7, in my search of the literature for linguistic scholarship regarding the word *just* in its sense of what I call *diminuendo*, I found nothing explicitly relevant. My search involved looking in places where I thought such work might appear, but I can never be sure that there is nothing. By contrast, if a consideration of the word *just* were present in the literature, it is extremely likely that I would have found it referenced in some or all of the places I looked. The scholarship that relates to this language practice should refer to other similar work. This is an example of looking for something that does not exist, but that might exist if a linguist had taken an interest in it. It is a greater challenge yet to look for or at something that could not possibly exist. This is a problem of languaging in mathematics. As the semiotics scholars in mathematics education have said, mathematics does not have concrete objects.

How do we express the non-expressive? How do we signify nothing? Rotman (1987) and Seife (2000) describe the Western world's struggle with the mathematician's desire to signify nothing. Indeed, nothingness is important in mathematics. Rotman and Seife look at the once-contentious signifier *zero* as an expression of nothingness. I want to turn that question around. Instead of nothing being signified by something, by the signifier *zero*, how does it work for something to be signified by nothing? What real things can be described by the absence of utterances, by silence?

Geometry and Silence

As a way of addressing this question about the realities that can be revealed by silence, we could look at silence's sister – empty space. Consider the white spaces in Walter's (2001) geometric reconstruction (p. 27) of Theo van Doesburg's painting *Arithmetic Composition I.*[17] Figure 8-1 shows these white spaces.

Figure 8-1. The white spaces in a geometric image

The white spaces are there, though no reader could be expected to discern them. It is much easier to see the white spaces in Walter's image when they are displayed in their context, in juxtaposition to the black, inked spaces.[18] Figure 8-2 shows Walter's geometric reconstruction with both black and white parts included.

Artists and designers refer to these empty spaces as "negative space." To expand on this terminology, silence could be called "negative language." Silence is only recognizable in juxtaposition with speech. This is the aspect of silence identified by Heidegger (1927/2003) and discussed in Chapter 4. A person's silence is not noticeable when he sits alone. Silence in the absence of context is similar to Figure 8-1, which looks just like a blank page. It is only in the context of a conversation that silence is significant. This is illustrated in Figure 8-2, in which the negative space is present for us because we have positive images for contrast.

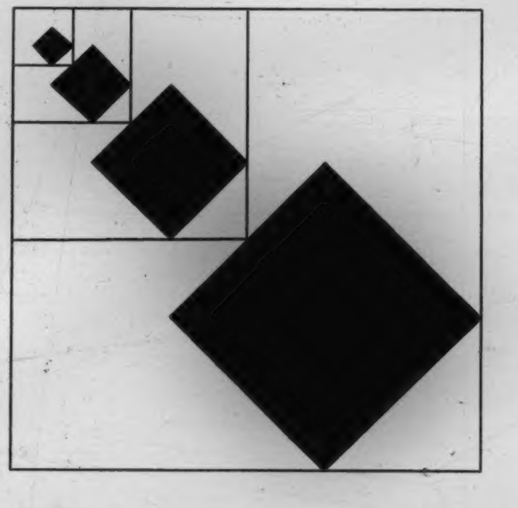


Figure 8-2. The full geometric image (Walter, 2001, p. 27)

In her popular how-to-draw book, *Drawing on the Right Side of the Brain*, Edwards (1999) describes how attention to negative space frees us to see objects as they are.

> Why does using negative space make drawing easier? […] [I]t's because you don't know anything, in a verbal sense, about these spaces. Because you have no pre-existing memorized symbols for space-shapes, you can see them clearly and draw them correctly. (p. 118)

In this account of my research, I have drawn attention to "positive language." However, this attention to literal utterances has exposed silences, the "negative language." Edwards' characterization of attention to negative space promises value in doing the opposite. It suggests that there is research potential in the study of the silences, as a way of seeing the discourse for what it is. The investigation of silence can be a technique for getting past our assumptions. With this technique we can see the edges of the discourse as we would not see them in studying the utterances themselves. It is a technique for seeing boundaries and definition.

Edwards (1999) shows how negative spaces are real: "[T]hey are not just empty 'air'" (p. 118). Silence too is much more than an absence. Silence itself speaks. When a person speaks, she closes off the possibility of other utterances that might have been spoken, by herself or by others. Thus speaking forms silence. Utterances reduce the landscape of the possible, because they begin to occupy the empty space that could be filled with anything. Sharp (2002) examined Walter's (2001) image and thought about how it might be done differently. Figure 8-3 shows one of his eight alternatives.

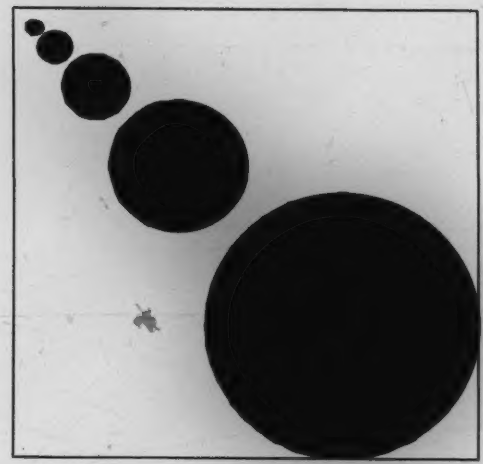


Figure 8-3. A variation of the image (Sharp, 2002, p. 19)

Though the glossing of a work of art as an utterance might be contentious, I will do so to support the connections between negative space and silence. When van Doesburg shaped his artistic utterance, he chose squares instead of circles, rectangles, triangles, hexagons or countless other possibilities. He silenced the other possibilities by choosing squares.

In addition to the silencing effect of choice, shapes that are not there in Walter's and van Doesburg's images can be inferred by a person who sees the images. Indeed, as Walter (2001) looks at the painting "with a mathematical eye" (p. 26), she sees other images that sustain or underlie the painting's surface. In her article she includes numerous geometric images with verbal descriptions. The line segments she inserts into the original image (Figure 8-2) represent explanations of the structuring and arrangement of the public image. These images are representations of the unspoken structure that shapes van Doesburg's utterance. They are images of his silence.

Critical Language Awareness and Silence

The above images from Walter (2001) and Sharp (2002) suggest that mathematics is a discourse that ought to welcome the investigation of possibility. Taking the words of Chouliaraki and Fairclough (1999) out of their context relating to critical discourse analysis, Sharp's investigation of alternate images could be said to represent a "range of possibilities [...] in given structural conditions" (p. 65). He asked, "What if the square were replaced with another shape?" as a way of understanding the actual, particular arrangement of squares.

I suggest here that critical language awareness (CLA) can be a means of listening to silence. Recall, from Chapter 3, Skovsmose and Borba's (2000) description of critical research methodology: "[W]e will suggest that doing *critical research* means (among other things) to research *what is not there* and *what is not actual*" (p. 5, emphasis theirs). Particularizing their description, critical research of discursive practice would suggest an investigation of silence, of the things *not actually said*.

For example, Chapter 5 describes a stream of conversation about the not-said references to particular humans. We started to think about how this not-saying related to what was actually said. This kind of thinking is similar to Walter's (2001) investigation of the van Doesburg painting. She was looking for imaginary images that structured and complemented the public image. Similarly, in discussing obscured or not-present human agency in mathematics utterances, we can look for the creative human beings who structure the public discourse of mathematics.

Chouliaraki and Fairclough (1999) say that the goal of critical discourse analysis is to become aware of a range of practices possible in a given discourse. Thus CLA, the pedagogical application of this stream of linguistic scholarship, seeks to educate students' awareness of possible ways of participating in their discursive space. Students who are critically aware of language would not wish to know about the particular form of their discourse as much as they would want to know how the form is *not*, and how it could be.

This question, "What if not?" is also a powerful tool for mathematical investigation. Brown and Walter (1990) demonstrate the power of this question when it is applied in a variety of mathematical contexts. The way of questioning that they advocate could appropriately be seen as an instance of "critical mathematics education" because of its parallels to CLA. Indeed, asking "What if not?" has proved to be an effective research tool in a number of disciplines. Brown and Walter (1993) edited a book in which various scholars describe the application of the what-if-not question to other fields. I suggest that when the question is applied in the research of discursive practice, it is called CLA. Critical language awareness involves the investigation of the language that is not there.

Chapter 9 – Cadenza: How Can Critical Language Awareness Be Done in the Mathematics Classroom?

It can't be done.

This statement – "It can't be done" – is familiar to mathematicians. Existence proofs have captivated many for centuries. For example, Andrew Wiles, in October 1994, proved a theorem that many others have investigated since 1670 – that there are no whole-number solutions to the equation $x^n + y^n = z^n$ for any power n greater than 2; it can't be done. The problem is commonly referred to as *Fermat's last theorem* (see Devlin, 1998).

In some sense, my research investigates an existence conjecture: Can critical language awareness (CLA) be done in a secondary school mathematics classroom? To address this question and show that it *can* be done, I needed to show at least one example of it being done, which is to show *how* it can be done. This is different from generalizing, which would involve listing *all* the ways it can be done, or describing in general what must take place in order for it to be done. Like Chapter 8, this chapter explores the space of the possible.

In this chapter, I address the question: How can CLA be done in the mathematics classroom? As cited in Chapter 2, Candia Morgan (1998) has suggested the idea of introducing CLA to mathematics students. And as I mentioned in Chapter 1, I talked with her in person about possibilities for CLA in mathematics classrooms. Her immediate response to my interest was to describe the response she had experienced from other mathematics educators who had read the book in which she broached the idea (Morgan, 1998). She said that these educators had been telling her, "It can't be done." This four-word sentence has haunted me throughout my research. It sounds so certain, like a sentence proclaimed by a magistrate. This sentence echoed in every silence that I confronted during my conversation with students, both their silences and my silences – silences that are detailed in Chapter 4.

What did these educators mean when they said it could not be done? To structure my response to the question of how CLA *can* be done, I will consider three different ways of seeing the assertion, "It can't be done."[19] First, it seemed clear that the educators of whom Morgan spoke were emphasizing the negation in the utterance: it *can't* be done. In response to their doubt, I will reiterate the kind of resistance that CLA can meet. Second, I wonder what the doubting educators were thinking about when they said "it." What can't be done? And, what *can* be done? I will ask what *it* is. What is the heart of CLA? Third, I will think about the verb *to do*. The form *done* suggests completion. Indeed, awareness cannot be *done*. It cannot be complete. It can only be prompted, educated, increased. After addressing the assertion that CLA cannot be done, I will outline an agenda for investigating further the possibilities for CLA in mathematics education.

It *Can't* be Done

I asked Morgan in our face-to-face conversation why people would say that CLA cannot be done in mathematics classrooms. We talked about two relatively obvious problems. First, we noted that in the United Kingdom and in Canada mathematics curricula are already overloaded with content. It appears that this problem is not unique to these two countries. It would be difficult to add anything to the scope of mathematics classes. Second, as with the introduction of any critical thinking, there is an element of resistance to present practices. We were aware of the potential for CLA to cause disruption in classrooms.

The problem of time is one that I felt in this research. Though I believed and continue to believe that time spent attending to language may well avert time-consuming problems in the mathematics learning, I was living in a dynamic relationship in which the predispositions of each partner in the conversation needed to be respected. I had to respect the students' and Mrs. Hill's senses of time well spent in order to maintain good relations with them. I could only push at the boundaries of their predispositions. I could not dash these boundaries to pieces without seriously compromising our relationship.

Any mathematics teacher who tries to spend class time attending to language is likely to face similar resistances. An educator would need to dance a dance of agency that is different from a student's dance. Instead of negotiating in real time the space between human creativity and the mathematics discipline, as students would do in a healthy classroom, a teacher would be dancing between creative exploration of pedagogical practices and old classroom traditions, which are the pedagogical discipline. For instance, in this research I needed to find a way of fitting attention to language into existing traditions, though there was no ready place to fit it in.

Though I had the complicating influence of another teacher, a person with considerable charisma and authority, I recognize that the students, collectively and even individually, are also major forces in any classroom. Walkerdine's (1988) account of a student named Archie (p. 51f), demonstrates the manner in which a student can control the classroom discourse. Similarly, in the context of clinical interviews, van den Brink (1982,1990) challenges the stereotypical attitude that interviewers do the observing and interviewees just present themselves as they are. He shows how interviewees also observe their interviewers, and suggests that it is important for interviewers to be aware of this. Both Walkerdine and van den Brink show how students can control what happens in a research situation that is supposedly controlled by the researcher. In light of their scholarship, students being asked to attend to language can be expected to resist oddities, especially in classrooms with a success-oriented, matriculation-intended student body. Such students can be wary of a teacher's allocation of time and other resources.

Time constraints are only a small part of students' resistance to oddity in the classroom. Introducing a new topic of conversation, such as language practice, and expecting a new form of conversation, such as critical questioning, push at the boundaries of the students' predisposition. If these boundaries are pushed too aggressively, students will not comply. They may actively resist, but, as I detailed in Chapter 4, this research has shown me that students can be especially skilled at passive resistance.

***It* Can't Be Done**

My conversation with Morgan continued by e-mail. When I reminded her that she had summarized the reaction to her promotion of CLA with a collective "It can't be done," she responded with a question about what *it* is: "Of course there are lots of further things to say and questions to ask. Perhaps most importantly: what is 'it'?" Indeed, attention to language can be prompted in many different ways. Adler's (2001) dilemma of transparency speaks of the on-going need to attend to language in any mathematics classroom, but there are different ways of doing so.

For example, Mrs. Hill typically introduced both new concepts and new general topics with a series of definitions. Would this be considered attention to language? It is.

But is it *critical* attention to language? This question strikes at the heart of CLA. I suggest that attention to definitions could be deemed “critical” if alternative definitions are considered. For example, in Lakatos’ (1976) fictional mathematics class, the participants’ consideration of various definitions of the word *polyhedron* was instrumental in their developing understanding of the Euler conjecture. Returning to the framework for critical discourse analysis put forward by Chouliaraki and Fairclough (1999), the goal is to become aware of a range of possibilities.

This kind of critical attention applied to language can form prompts for mathematical thinking. Hewitt (1999; 2001a; 2001b), in his series of papers about directing the awareness of mathematics students, suggests that teachers differentiate between the *arbitrary* and the *necessary* within mathematics. Language representations are largely arbitrary. Hewitt (1999) shows that the investigation of the necessary is the site of mathematical exploration, but also notes that certain ways of investigating the arbitrary draw attention to the necessary. I suggest that Brown and Walter’s (1990) what-if-not strategy for mathematical exploration exemplifies such exploration. What if we change a definition? (i.e. “What if not this definition?”) Or, what if we use personal pronouns instead of passive-voiced utterances in our mathematics communication? This is the question that underlies the stream of conversation in Chapter 5.

I assert that any attention to possibility rather than conventionality has the possibility of leading to language awareness. Recall Bakhtin’s (1953/1986) recognition that the tension between generally accepted meaning and individuals’ expressive meanings is at the heart of every discourse. One of my intentions for this dissertation, a complex utterance in itself, is to provide examples of possibility for mathematics teachers. I cannot expect any teacher to do the same thing that I did with my participant students. No mathematics teacher can be expected to have the same background as I. Teachers are busy people. Most teachers would not have the time to read about discourse analysis as extensively as I have, and teachers who would read extensively are unlikely to have the same intentions and interests as I had. Different conversations about language would emerge in their classrooms because of these differences and student differences in each class.

It Can't be *Done*

There are questions that can surface with a focus on the word *done*. If CLA is *done*, what is done for the mathematics students? What happens for students whose awareness is raised? Chapters 5, 6 and 7 suggest possibilities of what *can* happen for students. However, these were exceptional situations that do not typify the experience of all the participant students. In addition to the possibilities recounted in these three chapters, another thing was done for the students in this classroom.

In my interviews with students at the end of the term, a few of them articulated a sense that our attention to language had increased their comfort with mathematics in this class. For example, Matthew said, "It's just nice to understand the kind of stuff that goes on in your class, and stuff. It gives you a better feeling about what's going on in the classroom instead of just kind of being there." Brandon also suggested that he was drawn into discussion about mathematics by our attention to language: "I'm a little more intrigued about math, just this class."

I suggest that a critical awareness of language, an awareness of a range of possibilities for participating in the mathematics classroom discourse, gives students a sense of space, a sense of room to move. When it seems that there is only one path available, students can feel a sense of repression. By contrast, when they become aware of alternative ways of participating and positioning themselves within the discourse, there is a sense of freedom, of mobility. This expansion of the range of possibility can be initiated with even a little critical attention to language.

If, in the question, "How can it be done?", the "it" encompasses a wide range of possible practices that direct attention to various possibilities in language practice, then I suggest *it* can be done. Educators *can* raise awareness. However, awareness cannot be exhaustive. It cannot be complete. No one can be aware of everything – every point of view, every possibility. In this sense, it cannot be *done*. It cannot be finished. It can only be initiated. Educators can only aim to *raise* awareness.

An Agenda for CLA in Mathematics Education

A question for educators who are interested in raising awareness of language is *how* to do it and for what reasons. Because my researched classroom situation was different from regular classrooms in some significant ways, this dissertation's account of the situation cannot be translated blindly into just any mathematics classroom. In addition to my pedagogic reasons for wanting the students' critical awareness of language to increase, I had another agenda – the collection of data for research. In this way, my motives were different from those one would expect of a regular classroom teacher. However, I suggest two reasons why this difference is relatively minor. First, because my research agenda was based on my belief in the pedagogic value of CLA, my data collection agenda was closely related to my pedagogy. Second, any classroom teacher who tries something new, whether it is directing attention to language or something else altogether, probably looks for evidence of success or failure. In other words, the teacher aims to collect data.

The fact that the participant students were aware of my agenda was probably a more significant factor in what happened with CLA than were my mixed motives. As I noted in Chapter 3 and in my essay for the students entitled "What are we doing here?" (see Appendix B), the relevant ethics protocols required that the research participants be informed of the intent of the research at its outset. The students knew I was interested in language. Every time I mentioned the word *language* or drew attention to language in another way, students expected something odd to happen, something unique to their particular class and thus not necessary for learning the content of Pure Mathematics 20.

Their sense of oddity would change their perception of every task I gave them. By contrast, a teacher in a non-researched classroom could ask innocently of the students a question like, "What does that mean when Jessye says *just*?" In such a context, the students would be more likely to expect the question to lead to fruitful dialogue about mathematics. With this expectation, the conversation could develop more naturally.

I consider, for example, one class of Pure Mathematics 10 students that I taught in the year between my return from Swaziland and the beginning of my master's studies. This group of students somehow became interested in asking critical questions of the language and content in their textbook. Indeed, their fascination with critical thinking was probably instrumental in my choice to conduct the research described in this dissertation. Their critical questions often led to generative discussions about the mathematics in and around the prescribed curriculum. These students' questions and conjectures about the language choices of the textbook authors spun into questions about the nature of mathematics. Their awareness of the human agency involved in textbook-writing seemed to make them aware of the human agency behind the mathematics they were required to study. They became interested in the choices made by mathematicians and curriculum designers.[20]

For this class, discussion about language was natural. It was pure pleasure, probably because we all knew that it was not part of the agenda of anyone outside or inside the classroom. In the researched classroom, I often felt that the students' orientation to success (getting good grades) interfered with attention to language. However, the students were happy to be distracted from this agenda when the alternative was something they considered pleasurable. It is important that a teacher's prompts be perceived as authentic and for the moment. I can furnish this kind of prompt quite naturally because I have an interest in CLA and I recognize its potential for prompting reflection. However, in a situation coloured by ethical pre-clearance, such "natural" prompting is extremely difficult because there is an obvious externally motivated agenda.

Leading Children to Language Awareness

Thus, my first recommendation for educators who are interested in directing attention to language is to do so out of genuine curiosity and without reference to an external agenda. (Admittedly, it will be a challenge to follow this advice in the setting of a researched classroom because of the ethics protocols described above.) With this recommendation in mind, I will briefly describe a few settings in which I think such an approach would be worth exploring: with younger children, in a classroom with a more investigative (less traditional) culture, and with teachers. In addition to these settings, which differ from my research context, I believe critical attention to language is worth pursuing even in a relatively traditional secondary mathematics classrooms. Though only seven of the thirty-two students in this research ever became very engaged in discussions about language, and a further eight became somewhat engaged, I wonder how this proportion compares with the reaction of a typical group of students this age being asked by an adult to do anything. I believe directing students' attention to language can be particularly valuable if it can be done without the complicating factor of a clear external agenda.

Because younger children tend to be less concerned about "receiving" the curriculum content and because they tend to be more talkative with their teachers, I think conversations about language could flow more naturally with them. However, in the elementary grades, children's sense of language changes significantly. For example, my 5-year-old daughter is more innovative in her languaging than is my 10-year-old daughter. The elder daughter tends to have fixed ideas of what particular words and sentences mean, but when she was 5 her approach was more like her sister's is now. I expect that the 10-year-old will become more innovative again soon. Teachers or researchers interested in fostering critical language awareness in younger children need to be sensitive to their language development.

While younger children may be more inclined to talk about language, their receptive disposition does not necessarily suggest that critical awareness is more valuable for them. At the secondary level, children often become aware of their emerging place in their world. I believe that during this time it is critical for them to become aware of agency and voice, despite the obstacles their teachers might face. My conversations with the students in the researched classroom have made me more convinced of this. I am still interested in pursuing CLA with such children.

In a secondary school mathematics classroom, I think CLA would seem much more natural if the classroom culture were more investigative. If students were regularly given tasks that demand creativity and innovation, there would be more language flowing and more dancing between human and disciplinary agency. In this kind of setting, a setting significantly different from the relatively traditional class I researched, questions about language practice are more likely to seem relevant to the mathematics.

Leading Teachers to Language Awareness

In addition to directing students' attention to language, it is important to consider how to raise teachers' awareness of language. I believe that the best way to do this is to engage them in mathematics and to direct attention within this context to their language practice, just as I would with children. This could be done in various settings, including in-service workshops for practicing teachers and courses for pre-service teachers and graduate students. If teachers are not critically aware of their language practice in mathematics communication, they are unlikely to be interested or able to prompt their students' awareness.

There are many ways in which an educator can direct teachers and emerging teachers in experiences of mathematics, and also many ways to direct attention to language within these experiences. Here I present one approach that I plan to implement in the coming year in a course for undergraduate students learning to teach mathematics. The approach does not focus on language. It focuses on mathematics. I suggest that this prioritization is fundamental.

Many of my undergraduate students say that they have never "taught" before. So I like to have students do a practice lesson in my mathematics teaching methods courses. Unfortunately, there are time constraints. These practice lessons have to be very short. And with a short time allotment, it is very difficult to engage others in mathematical thinking. Even students who try very hard to do more than "transmit knowledge" find themselves hurrying to get information out instead of dwelling with their pretend students in mathematical conversations.

In the course that I have in mind, the first assignment will have students do a mathematical investigation together with another person – a friend, a relative, another student, anyone. I will give each student a collection of mathematics problems. My student, along with his or her partner, will select one and do it without looking at it in advance. Students will videotape themselves working on the task with the other person.

They will be asked to submit to me a written reflection on their experience of doing the mathematics and on what they have seen as they watched the video record. As part of this reflection, I will ask them to choose a small but significant portion to transcribe. They will not submit to me their videotape, only the reflection and the transcript of part of the mathematics work. They will use their transcript to exemplify their reflections on doing investigative mathematics (which is in fact *learning* mathematics).

I will not direct my students in advance to look at the language in the transcript, even though it is an obvious interest of mine. However, in my written feedback to them, I might point out language features that I notice as they relate to the issues they raise in their reflections. I will ask these students questions about these language features, features that I see as being significant in mathematics and in the teaching of mathematics.

I will refer back to their reflections on the video-taped mathematics throughout the course – in my responses to their journals and other work, and also in class discussion. The final assignment will be the same as the first, except that the students will do the same mathematics task with someone who has not done it before. This situation will resemble that of teaching. My student will "know" a solution, but she or he will be working with someone who does not "know" it. What does the one who knows do to promote the mathematical thinking and reasoning of the one who does not know? And what can the knower learn from drawing out the reasoning of someone who comes at the task from a different perspective? This is the situation teachers face all the time. I will be interested to see if my questions about language will impact their second experience of the mathematical investigation and their reflection on it.

It Can be Done

Although in this chapter I have given some advice for mathematics educators who are interested in raising their students' awareness of language, I reiterate that every teacher should attend primarily to his or her students and to the mathematics being done by these students. Attention to language can *support* the learning of mathematics, and can especially support an understanding of what mathematics is. I am promoting a little attention to the nature of the mathematics class discourse and some investigation of possible alternative forms within the discourse. After a teacher seeds critical attention to language practice, students have the opportunity to apply the same approach to other language in their mathematics classroom and elsewhere. It can be done.

Chapter 10 – Coda: Voices that Invite Resonant Silence

> [S]ilence is the ground from which all speech emerges and into which it falls back. (Bollnow, 1982, p. 41)

After the close of my interaction with the participants in the researched classroom, I asked: What was this all about? What was central to the discourse that we developed together, beyond the expected and conventional classroom mathematical discourse? I concluded that among other things it was about silence, silence in various forms.

Silence is a part of any discourse. It is a part of any utterance. As long as a person remains literally silent, the choices of what to say are infinite. As soon as she speaks, she chooses from this infinite domain. With this choice, she closes off all other possibilities of things that she might have said. Even the most talkative people leave more unsaid than said.

Language might be seen as the surface manifestation of the silences that sustain it. Consider, for example, the face of a familiar person. We know our friends by their faces, but any face is the mere surface of all that is behind it. We see the outer layer of skin, but this is not the person. We might argue that we know more from the friend's actions or words. But these words are also a mere surface of the person's thoughts.

Or consider the moon. What do we see of it? Only half of its surface. But that is not the moon; it is only part of it. The moon is made up of its surface as well as what lies beneath the surface, which is inaccessible. As noted in Chapter 4, Neumann (1997) suggests that it is silences that sustain the spoken aspects in a discourse. The silences lie underneath, upholding the surface that is said and heard.

This thesis is an account of an on-going conversation that included thirty-two students, their teacher and myself. Like the surface of the moon, this writing is the primary public face of something substantial that lies behind it – the complex discourse in the researched classroom. The account could be written differently with attention to other aspects of the conversation. This extended utterance is the result of countless choices, some of which I mentioned in previous chapters. Other choices have not been mentioned, like the decision to direct attention to the participant students' voices more than to the participant teacher's voice.

In the fourth chapter, I reflected on different kinds of silence and characterized three categories – silences that speak when they are juxtaposed with expected utterances, self-censured silences and externally censured, repressive silences. These categories are not mutually exclusive. The extent to which a silence speaks relates to the extent to which it is noticed in a conversation. The other two categories seem to form a dichotomy – the internally chosen silence versus the externally enforced silence. However, because we can choose whether to comply with externally enforced silencing, even repression cannot be separated completely from individual choice.

While it is possible to speak more clearly with silence than with a literal voice, some of our silences are not heard. Such silences suppress potential conversations by failing to initiate them. The suppression of particular conversations might be a result of individuals choosing to be silent or of voices being externally repressed, either explicitly or implicitly through the agency of the discourse.

Bollnow's (1982) list of different types of silences is longer than mine. It includes brutish silence, defiant silence, icy silence, secrets, diplomatic silence, silence by compact, comprehending silence, wordless agreement, and silence of awe. Some of his categories fit into my three categories. For example, defiant silence, in which "[t]he person is using silence to shut himself off from the outside world" (p. 43), is an example of a silence that speaks. Bollnow's icy silences, diplomatic silences, silences by compact and wordless agreements can also fit into my category of silences that speak. His list has a different nature than mine does.

All of his silences, but for one, seem to be silences chosen by the person who holds them. None of them is a silence imposed by an external force or discipline. However, one of Bollnow's (1982) categories of silence is significant, as it stands apart from the others in his list and in mine – the silence of awe. It is not exactly a self-chosen silence because it suggests a sense of compulsion. And because the silence is not sustained by a person's or a community's surrender – either willing or unaware – to disciplinary agency, it is not externally suppressed. There is only the subconscious realization that nothing should or perhaps could be said. It prevents a person

> from revealing his innermost life or from interfering tactlessly or from sheer curiosity in another's sphere of intimacy. It is for this reason that we usually fall silent in the presence of the holy, in church, in a temple or in the religious sphere in general, in the same way as we fall silent in the presence of nature in her overwhelming grandeur. We dare not speak aloud. (p. 44)

Sinclair and Watson (2001), as they wonder about awe in the mathematics classroom, seem to be addressing this kind of silence. Watson notes that in the United Kingdom, mathematics teachers are expected to contribute to their students' spiritual development. Together with Sinclair, she explores opportunities to promote awe in mathematics. They relate awe to wonder and suggest mathematical exploration as a site for wondering. They add that in addition to the knowledge that makes such wondering possible, the "capacity to wonder also involves a confession of limitation or ignorance. To wonder is to acknowledge one's ignorance, not in a state of despair or passivity, but in the pleasurable pursuit of further knowledge" (p. 40). Their suggestion that wonder involves quietly accepting one's ignorance lends a further example of Bollnow's (1982) silence of awe.

This kind of silence is one that I felt at times in response to my on-going conversation with the students and teacher in my researched classroom. Together with these participants, I wondered about possibilities within the discourse. In my interpretations of our conversation, I continued to wonder about these possibilities and their connection to aspects of mathematics teaching and learning identified by other scholars. However, this exploratory *wondering* is accompanied by a silent awe. I stand in wonder at the complex weave of many voices that any mathematics class comprises, at the volume of texts that find their way into the mathematics class and at the incomprehensible manner in which meaning arises from shared mathematical and everyday symbols.

Bollnow (1982) characterizes a good conversation as one that sinks into awed silence.

> [W]hen the conversation finally does sink into silence, it is no empty silence, but a fulfilled silence. The truth, not only of the insight that has been acquired, but the truth of life, the state of being in truth that has been achieved in the conversation, continues to make itself felt, indeed becomes deeper, in the course of this silence. (p. 46)

This is how good conversations end, with a silence that respects the shapelessness of the experience. As long as we speak about the experience, our accounts shape it and transform it into something else. A silent conclusion to a significant experience allows the experience to eclipse the conceptualization that is a natural part of speaking. Bollnow suggests that these shapeless forms in which conversations end also beckon new conversations:

> [W]hen I said that speech fades away in nothingness […] [i]t is indeed a nothingness, but a nothingness in the deeper sense in which Japanese thinkers have used this word, nothingness as the shapeless primal ground that is struggling for shape and from which all shaping emerges. (p. 46)

Bai (2002), a Buddhist scholar, describes how naming an experience by speaking or writing about it arrests the experiencing. Like Bollnow, she writes about the tension between speaking and remaining silent, between conceptualization and contemplation.

> [W]e are so caged up in our thought constructs that we have difficulty realizing that these are *just* our thoughts, *just* constructs of reality. This is not to be dismissive of thought constructs. They are the very substance of our world-making, and world-making is human beings' particular and proper way to inhabit and work with reality. But when we are caged up in concepts and are driven by them, we do not have the freedom to make worlds as we see fit. (¶38, emphasis hers)

Speech falls to silence, whether it is awe-filled silence, a silence that rests on a sense of exhaustion or completion, or one that is enforced by an exterior time constraint. But awe-filled silence, the silence in which we marvel at the inadequacy of words, begs new conversations. It invokes conceptualization. Because awe-filled silences follow significant experiences, we are drawn to make connections between these experiences and previous experiences that were especially meaningful.

Michael Balint, a psychoanalyst, also recognizes silence as much more than nothingness. He encourages therapists to listen carefully to their patients' silences.

> The pedestrian analytic attitude is to consider the silence merely as a symptom of resistance to some unconscious material stemming either from the patient's past or from the actual transference situation. One must add that this interpretation is nearly always correct; the patient is *running away* from something, usually a conflict, but it is equally correct that he is *running towards* something. (Balint, 1968, p. 26)

My Chapter 4 in some way represents the pedestrian attitude Balint describes. I took the students' various silences as a symptom of their resisting attending to language in mathematics class. However, in that chapter and the following ones, I further explored the nature of this silence. I have come to see the students' silences regarding language and critical attention as an aspect of their participation in their community of mathematics learners (which is different from *the* community of all people who do mathematics). They were expressing their complicity in a discourse that tends to mask or suppress human agency.

Here Balint's encouragement is significant. If the students in this research were running *away* from human agency, what were they running *toward*? It would seem that they sensed themselves being encouraged to run toward a way of doing mathematics that follows preset paths (cf. Alrø and Skovsmose, 1996). However, there are alternative ways of living within a discourse whose members run away from expressing human agency. As

Balacheff (1988) underscores, depersonalisation is part of the nature of mathematics. The absence of particular human references in certain mathematical utterances is part of what characterizes mathematics. This absence is the negative space that surrounds the shape of mathematics. One way of finding human agency within this discipline is to draw attention to the role of individuals in composing utterances that are true generally (and thus independent of personal particularities). Though depersonalisation is a necessary aspect of a generalization, there is always a person making the utterance.

With careful attention to the silences in the mathematics classroom we can learn something about the discipline of mathematics and about children's sense of relationship with this discipline. What do they fear? What else, besides expressions of human agency, are they running away from? And then, what are they are running towards? As Balint notes, mathematics is a natural ground for silence; mathematics, philosophy and creative disciplines, such as the arts, share this common ground. In these disciplines "there is no external object present, and thus no transference relationship can develop. Where there is no transference, our analytic methods are powerless" (p. 24). Other methods are necessary for serious consideration of silence. Balint suggests that attention to silence in these disciplines should help us understand the person's relation to these disciplines themselves: "Perhaps, if we can change our own approach from that of considering silence as a symptom of resistance to studying it as a possible source of information, we may learn something about this area of the mind" (p. 27). This has been my contention throughout this dissertation.

With this exploration of language, I have sought to look at something that actually exists – secondary school mathematics classroom discourse. In the exploration, I developed an interest in things that are not there, the silences endemic in the discursive practice of the classroom. Following the silent wonder that initially marked my experience of the extended conversation with the participant students and teacher, I have been moved to speak about that silence. I am beginning to wonder about possible ways of researching silence and voids in the mathematics classroom. In this study, my awareness of silences *emerged* from the experience. How might we enter an experience with the *intention* of listening to silence?

Endnotes

1 The differences between pure and applied courses in Alberta, the province in which this research was conducted, include more factors than the differing natures of pure and applied mathematics. Local universities' admission requirements favour the pure over the applied programme. This preference directs students who intend to matriculate to take the pure stream of courses. Thus, a "Pure Mathematics 20" class might be characterized alternatively as an "academic" grade 11 class.

2 "Pure Mathematics 10" is a course designation in Alberta's program of studies for high school mathematics (Alberta Learning, 2002).

3 "Pure Mathematics 20" is a course designation in Alberta's program of studies for high school mathematics (Alberta Learning, 2002).

4 An example of this tension between public definition and personal meaning based on experience can be found in Alcock and Simpson's (1999; 2002) reports on the difficulties students experience in the move toward definition-based understanding in undergraduate mathematics.

5 I report this conversation with no quotations because I have no audio record of it, and I did not record people's exact words in my field notes at this moment. I consider it appropriate practice to reconstruct the utterances in a dialogue in certain situations, but in this dissertation I do not do so unless I explicitly say that I am doing it. I want to be clear about which utterances are recorded (by hand or by video- or audio-tape) and which are recalled through memory.

For the first three weeks of class, I had no recording equipment on hand because I was waiting for the students' informed consent. After this time, there were situations in which the recordings did not work – when the recording devices were not on (before and after class) or when the conversation was beyond the range of the recorder. However, I made field notes every day. Sometimes I hand-recorded excerpts of conversations.

6 Actually, I am aware that my treatment of silence in general and my analysis of silences in this research may begin to push these boundaries. However, I am not pushing the boundaries far. Research done following the narrative inquiry model put forth by Clandinin and Connelly (2000) pushes harder at some of the traditional boundaries that form academic discourses.

7 Because this conversation was not mechanically recorded (I was awaiting clearance relating to the required ethics protocol), I rely on my field notes for this account.

8 It may be significant that some of the more animated discussions about language in this classroom were marked by laughter. This discussion is an example. Bakhtin (1936/1984), in his analysis of Rabelais' work, underscores the importance of understanding the role of laughter in interpreting utterances. I do not claim to understand the nature of the episodes of laughter during my conversations with the students and Mrs. Hill in this research. However, I recognize that this laughter was significant. Laughter appears often in the transcript excerpts in this thesis.

9 After my time with the researched class was over, I asked some Russian-speaking friends about the name *Signot*. They do not recognize it as a name nor as a Russian word. The Russian-English dictionaries I have checked do not have the word *signot* either. However, in my internet searches, I have found that some people have the given name *Signot*. From the contexts in which the name appears, I cannot discern if the people are Russian. Nevertheless, the boy whom I call Signot expressed a rationale that suggested his interest in anonymity as a concept. I suggest that his interest in this concept is more important than the veracity of his assertions about the name.

10 Though Pickering (1995) rejects the possibility of material agency in mathematics, I think that the question should be pursued: What is the nature of material agency in mathematics?

I think, for example, of Bloor's (1994) philosophical investigation of 2 + 2 = 4 proofs. He considers a naïve proof in which people who would "say, 'Look, here are two apples, one, two'. Then they take another two apples and say, 'one, two'. Then they bring the two pairs together, and conclude by counting them, 'one, two, three, four'" (p. 23). Bloor responds: "All the naïve proof does is to produce a truth about four apples, rather than establishing a timeless necessity about the number 4" (p. 23). It seems from this assessment that at a naïve level, there is material agency in mathematics. "2 + 2" surrenders to the agency of materials at hand, whether they are apples or dinosaurs.

I suggest that the question of material agency is significant for mathematics educators because students tend to have the naïve perspective that

Bloor describes. For example, in this researched classroom, Brandon responded to the question: What do you think about opinion in mathematics? He said, “Take something like ‘five plus nine’. In my opinion, it’s seven, but it’s gotta be more than nine. But I’m saying, that’s not true is it?”

Further to the question of material agency in mathematics, Namukasa (forthcoming) interprets children’s mathematical investigations from an enactivist perspective. She ascribes agency to the concrete manipulatives the children had at hand as they engaged in their exploration.

11 Student answers to a questionnaire at the end of the term show an even more depressing lack of personal agency. One of the prompts on the questionnaire was: Please describe a situation in which you felt that you contributed something unexpected or special in a mathematics discussion. Figure 11-1 shows their responses.

Student	*Please describe a situation in which you felt that you contributed something unexpected or special in a mathematics discussion.*
Arwa	never happened
Brandon	I bring the comedic aspect.
Darren	None, or can’t remember at all.
Donald	When the teacher asked me a question and I told the answer, or when the teacher asked a certain question to the class and I answered it. I blurted out an answer. I corrected the teacher.
Jessye	I can’t really think of a time. Sorry!
Joel	Not in a long time
Kalli	me, my presence … teehee dunno, never really talked in any of the discussions … too shy
Laura	I can’t really think of one, but if I ever did contribute something, I probably didn’t feel it was unexpected or special because most any answer I’d give could have been given by someone else.
Lisa	Can’t think about one!

Mansoor	When my friends and I were studying one of the math questions and we were all helping each other when we got stuck on one about tangent and I mentioned that one of the angle looks like a right angle and then my friend gets how to do the questions and say thanks when I didn't even know what I was doing. It was weird.
Matt	Sometimes I look at questions in a different way from some people or take my own approach and short cuts to solving it. Sometimes when I share these with the class, some people are completely confused, and other people understand it.
Matthew	I can't remember a time exactly.
Neeta	Sometimes shortcuts of doing things come really clear to me and I can be good at finding them.
Priscilla	never happened
Raina	I can't think of any time.
Rory	refer to Kalli's [*This is what Rory wrote.*]
Signot	Well that test thing I did individually, where Mr. Wagner thought that my reasoning for short answer and multiple choice questions would be different. I think he didn't expect me to think the same way on them.
Sita	Never had one
Tharshini	Can't remember!
Trina	I cant remember it happening.
Zamdar	On the first day of grade 6 I got my Appendex taken out. But before I did. I spewed all over my desk in math.

Figure 11-1. Students' recalled expressions of personal agency

12 This task, which I called "obstinate pain," was inspired by Watson's (1994) description of a task with which she has engaged her students. Students stand around a particular student and give instructions, telling that student what to do to walk the path of a three on the floor. The student is supposed to do *exactly* what they say, no matter how wrong the result would be.

13 The way a person directs his or her gaze is also influenced by cultural particularities. Gender and ethnicity, for example, are significant factors.

14 The adverbs *just* and *simply* are not technically diminutives but they serve a similar function.

15 Grice's five principles are listed below. This list is only partially published by Grice, having been first delivered in a lecture, but widely described by other linguists (e.g. Levinson, 1983).

1. *The Co-operative Principle*
   make your contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged.
2. *The maxim of Quality*
   try to make your contribution one that is true, specifically:
   - (i) do not say what you believe to be false
   - (ii) do not say that for which you lack adequate evidence
3. *The maxim of Quantity*
   - (i) make your contribution as informative as is required for the current purposes of the exchange
   - (ii) do not make your contribution more informative than is required.
4. *The maxim of Relevance*
   make your contributions relevant
5. *The maxim of Manner*
   be perspicuous, and specifically:
   - (i) avoid obscurity
   - (ii) avoid ambiguity
   - (iii) be brief
   - (iv) be orderly

(Levinson, 1983, pp. 101-102)

Gerofsky (1996, 2004) considers mathematical word problems as a genre and shows how the typical word problem flaunts the Gricean maxim of Quality.

16 Though I generally avoid gender-exclusive language, I present the "ideal soldier" as male. As I describe the gendered nature of mathematics (its maleness),

I compare it with the ideal soldier, who also exhibits maleness. It seems to me that the martial establishment is organized around male values. I do not want to say that women cannot make good soldiers, but I admit to being bewildered in this regard. I cannot comprehend what characteristics make a soldier "good."

17 A reproduction of the van Doesburg painting appears in Baljeu (1974, p. 99).

18 With the real painting, it would be easier to see the "silent" spaces. If we were to show a copy of the light spaces in the painting and leave out the dark spaces, there would be something to see because van Doesburg *painted* the light spaces. His brush marks even show a little colour.

19 I copy this structuring framework from an article in which Davis (1995) answers the question, "Why teach mathematics to all students?" with four different answers: "*you* need it," "you *need* it," "you need *it*" and "you-need-it."

20 Clearly, this recollection of a teaching experience from four years ago is coloured by my intervening experiences, including that of the researched classroom. For example, my word choice in this account of that grade 10 class experience is new. The words in this account were not part of my vocabulary at the time. Nevertheless, it is clear to me that those students were critically attentive to language, at a more naïve but more engaged level than the students in the researched classroom.

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Appendix A

[first day of classes]

Dear Ms. [Hill]:

I invite you to participate in a research project in the upcoming school term. The purpose of the research is to listen to students' experiences of becoming aware of their language practices in their mathematics learning. Data from this study will be part of my doctoral dissertation and may contribute to my university teaching and to articles, books and/or presentations for teachers and teacher educators.

This research would involve you and me in collaborative lesson planning for a particular Pure Mathematics 20 class. I would be present for most of the times that the class meets, and I would teach the class at various times throughout the term. We would plan to direct students to become more aware of the way language is used in mathematics by engaging them in dialogue and reflective writing about the communication in their own mathematics class. Such attention to language in mathematics is innovative and can be expected to aid the students' understanding of the Pure Mathematics 20 curriculum, and our understanding of their experience of mathematics learning.

Not only would classes be video- and/or audio-taped, but I would be asking your students to allow copies of their mathematics work and reflective writing to be made. I will also be making observational notes and transcribe selected dialogue. Both of these kinds of records will be shared with the participants in the research, you and participant students. Besides our collaborative planning, I would also ask you to have regular debriefing sessions with me, in which we discuss the experience of heightening language awareness. These sessions would be audio-taped.

We would invite students' parents to a meeting in order for them to learn more about this research project. At the meeting, I would explain the project and their children's prospective involvement in it. Students who do not wish to participate in this research from the outset would be reassigned to another section of Pure Mathematics 20.

At any time, participating students would have the option of withdrawing their participation permanently or temporarily. They will be given the chance to confirm or withdraw involvement on a regular basis through the course of the research. Audio and video tape recordings that involve a non-participating student will be destroyed and I will not take notes on these students. Should you choose to opt out or withdraw at any time I will stop conducting research in your class. You may also choose to withdraw your consent for the use of any data after its collection. If you are involved in a videotaped episode that I would like to include in conference presentations, I will request your informed consent based on a screening of the video segment.

If you agree to participate in this study, anonymity will be maintained through the use of a pseudonym. The name of your school and district will not be identified. Only my faculty advisor and I will know your identity. If you choose to co-author papers or co-present workshops and presentations, you must do so with the understanding that you give up anonymity. Data collected during this study will be secured in my office and any identifying information will be removed.

Please call me (492-3760) or my advisor Dr. Elaine Simmt (492-0881) with any question you might have. Consent forms will be required from all participants in this study.

Sincerely,

David Wagner

This study has been reviewed and approved by the Research Ethics Board of the Faculties of Education and Extension at the University of Alberta. For questions regarding participant rights and ethical conduct of research, contact the Chair of the Research Ethics Board at (780) 492-3751.

[first day of classes]

Dear Parents:

Your child's Pure Mathematics 20 teacher, [Ms. Hill], has agreed to participate in a research project in this school term. The purpose of the research is to listen to students' experiences of becoming aware of their language practices in their mathematics learning. Data from this study will be part of my doctoral dissertation and may contribute to my university teaching and to articles, books and/or presentations for teachers and teacher educators.

This research will involve your child's teacher in collaborative lesson planning with me. I will be present for most of the times that your child's Pure Mathematics class meets, and I will teach the class at various times throughout the term. The teacher and I will plan to direct students to become more aware of the way language is used in mathematics by engaging them in dialogue and reflective writing about the communication in their own mathematics class. Such attention to language in mathematics is innovative and can be expected to aid your child's understanding of the Pure Mathematics 20 curriculum.

Not only will classes be video- and/or audio-taped, but I will be asking your child to allow copies of his or her mathematics work and reflective writing to be made. I will also be making observational notes and transcribe selected dialogue. Both of these kinds of records will be shared with the participants in the research, the teacher and students. If your child is involved in a videotaped episode that I would like to include in conference presentations, I will request your informed consent based on a screening of the video segment. Occasionally your child may be asked to grant an interview 15 to 45 minutes in length. In this event, I will notify you in advance with a letter.

You are invited to a parent meeting on Thursday, [third day of classes] at 7:00 in room [Ms. Hill's room number], [name of high school] for learning more about this research project. At the meeting, I will explain the project and your child's prospective involvement in it. If you are unable to attend this meeting, information will be sent home following the meeting. Students who do not wish to participate in this research from the outset will be reassigned to another section of Pure Mathematics 20.

At any time, you or your child has the option of withdrawing his or her participation permanently or temporarily, or to withdraw consent for the use of any data in which he or she was a participant. Your child will be given the chance to confirm or withdraw involvement on a regular basis through the course of the research. Audio and video tape recordings that involve a non-participating student will be destroyed and I will not take notes on these students. Should the teacher choose to opt out or withdraw at any time I will stop conducting research in this class.

If you agree to participate in this study, anonymity will be maintained through the use of a pseudonym. The name of your school and district will not be identified. Only your teacher, my faculty advisor and I will know your identity. If Ms. [Hill] chooses to co-author papers or co-present workshops and presentations, there is the possibility that the school and your class could be identified. However, this probably will not lead to revealing the identities of particular students. Data collected during this study will be secured in my office and any identifying information will be removed.

Please call me (492-3760) or my advisor Dr. Elaine Simmt (492-0881) with any question you might have. Consent forms will be required from the parents of all students who will participate in the project class.

Sincerely,

____

David Wagner

This study has been reviewed and approved by the Research Ethics Board of the Faculties of Education and Extension at the University of Alberta. For questions regarding participant rights and ethical conduct of research, contact the Chair of the Research Ethics Board at (780) 492-3751.

Research Consent Form – Teacher

I, ____________________________, hereby agree to allow David Wagner to use my class as a site for his research on students' experiences of language awareness in mathematics learning.

I hereby consent:

- to be video- and/or audio-taped in the mathematics class;
- to interact with the researcher in the mathematics class;
- to interact with the researcher in planning class activities;
- for my work (plans, handouts, assessment instruments) to be photocopied for the purpose of the research.

I understand that:

- I may withdraw from the research at any time without penalty;
- all information from which I can be identified will be treated confidentially and discussed only with Mr. Wagner's supervisor;
- I will not be identified in any documents resulting from this research unless I choose to co-author papers or co-present educational sessions;
- my work may be scanned and included in research reports, published articles and books.

I also understand that the results of this research will be used only in the following:

- Mr. Wagner's doctoral dissertation;
- presentations for educators and others concerned with educational matters;
- written articles and/or books for educators and others concerned with educational matters.

____________________________	____________________________
Signature of Teacher	Date Signed

For further information concerning the completion of this form, please contact:

David Wagner		Dr. Elaine Simmt	
phone:	492-3760	phone:	492-0881
email:	drwagner@ualberta.ca	email:	elaine.simmt@ualberta.ca
post:	341 Education South University of Alberta Edmonton, Alberta T6G 2G5	post:	341 Education South University of Alberta Edmonton, Alberta T6G 2G5

This study has been reviewed and approved by the Research Ethics Board of the Faculties of Education and Extension at the University of Alberta. For questions regarding participant rights and ethical conduct of research, contact the Chair of the Research Ethics Board at (780) 492-3751.

Research Consent Form – Parent(s)

I, ______________________________, hereby consent for ___________________________ to participate in Ms. [Hill]'s mathematics class, which is a site for research into students' experiences of language awareness in mathematics learning.

I hereby consent for my child (please check the boxes):

- ☐ to be videotaped in the mathematics class;
- ☐ to be audiotaped in the mathematics class;
- ☐ to be interviewed;
- ☐ to allow his/her work to be photocopied for the purpose of the research.

I understand that:

- my child may withdraw from the research at any time without penalty, and, in the event this happens, my child will continue to be taught the mathematics content with his or her classmates;
- all information gathered will be treated confidentially and discussed only with Mr. Wagner's supervisor;
- my child will not be identifiable in any documents resulting from this research (If Ms. [Hill] decides to co-author articles or presentations with Mr. Wagner, there is the possibility that the school and participant class may be identified);
- my child's work (without his/her name) may be scanned and included in research reports, published articles and books.

I also understand that the results of this research will be used only in the following:

- Mr. Wagner's doctoral dissertation;
- presentations for educators and others concerned with educational matters;
- written reports, articles and/or books for educators and others concerned with educational matters.

______________________________ ____________________
Signature of Parent/Legal Guardian Date Signed

______________________________ ____________________
Signature of Participant Student Date Signed

For further information concerning the completion of this form, please contact:

David Wagner		Dr. Elaine Simmt	
phone:	492-3760	phone:	492-0881
email:	drwagner@ualberta.ca	email:	elaine.simmt@ualberta.ca
post:	341 Education South University of Alberta Edmonton, Alberta T6G 2G5	post:	341 Education South University of Alberta Edmonton, Alberta T6G 2G5

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Appendix B – Shared Writing

1. Rigorous Pedagogy (week 6): I wrote this and showed it to Mrs. Hill in the sixth week of the study. We discussed it in an interview. Her reaction demonstrated to me that she misunderstood my criticisms. I thought I was criticizing myself for my troubles drawing students' attention to language. She thought I was disgruntled with her and our relationship. I believe we cleared up this misunderstanding.

Rigorous Pedagogy

It think I've come to a crossroads in the research. I've noticed that I have been feeling uncomfortable with my initial approach to language awareness in the mathematics classroom. I have become less and less willing to lead the class in discussions about transcripts or about features of language. Now I think I know why.

My choice to explore language awareness in a classroom environment, rather than a clinical environment was significant in that it demonstrates my interest in pedagogy, in the practice of instructively walking beside children in the places they walk. And in our society, the place children primarily walk is school. I chose to research in a real classroom because I am interested in the teaching as well as the learning of mathematics.

With my research, I planned to insert language awareness activities into the classroom. This insertion was rooted in my conviction that there would be pedagogical value in doing this. But I am seeing now that this kind of insertion is not unlike a clinical setting. It is artificial in a number of ways. No wonder that the students seem to treat the language awareness activities we have done as something extra-curricular, outside their main responsibility!

I realize that the pedagogue in me has resisted the apparent interruption of our mathematical journey to consider something else, even though this 'something else' has potential for bringing richness to the mathematical journey. I believe that my pedagogic spirit, which drives me to do certain things and resists some of my intentions, has been bolstered by my observation of Cheryl [(Mrs. Hill)], in whom I see a strong pedagogic spirit as well. Teachers, researchers and other experts may argue about the value of her approach to teaching mathematics (and about my approach), but I doubt if anyone would deny that she strongly cares for her students. She is a rigorous pedagogue, and I hope that I am one too. Though 'rigorous' is a strong and cold word to describe caring, I want to use it in the same sense that Max van Manen uses the word to describe good phenomenological writing – not hard and unmoving but devoted and focused on a particular thing. A rigorous pedagogue is a person who focuses on children's needs.

What then does this say about my research project? How should it be transformed? I no longer want to see myself inserting language awareness activities into the classroom. Instead, I want to see myself thinking pedagogically. However, a language awareness component remains, because I myself am language aware. I have language analysis skills, among other skills, in my pedagogical tool belt.

During the course of class discussion, whether I am teaching or Cheryl is, I notice things that relate to language. When this happens, I can draw on my language analysis skills to direct students' attention to important issues in their mathematics learning. Instead of doing language analysis, I would be 'doing teaching' and drawing on language analysis to aid in that endeavour.

So, this is what I propose for the immediate future. I will not plan special language analysis sessions. Rather, I will be attentive in class and prompt discussion about things that just happened in the discourse. When I teach, I might prompt this discussion immediately. When Cheryl teaches, I will wait for an appropriate time, like the end of class.

There are a number of practical advantages to this. When I have shown students transcripts and video, they could not remember the events in the record. With my new approach there will be more immediacy. Students should be able to recollect the events under discussion. Also, I expect these discussions to seem to be part of the 'regular' mathematics learning, because they will relate directly to the mathematical topic at hand.

Up until now, I have not been sure what to ask interested students or Cheryl in interviews. With this new approach, I think there will be something to talk about. I can play the video of one of these emergent discussions and ask them what they think of it. "What did this discussion do for your (or the students') experience of the mathematics?"

2. A Sample Anecdote (week 13): When I asked students to give me accounts of instances in which they said or thought with the I-voice, some of them complained that they did not think with words when doing mathematics. In response to these students' assertion, I assigned the class the task of writing for me an anecdote of a situation in which they could not find words for what they were thinking. Because they were having trouble writing these anecdotes, I gave them an example: I wrote the following anecdote of such an instance from my experience teaching them. I shared it with them in the thirteenth week of class and asked for their response. My aim was to help them write their own anecdotes.

On the phenomenon of not being able to verbalize something I know.

I thought I was finished explaining the solution of a circle geometry problem. "*x* is equal to two root five," I say in closing. "So how long is the chord?" I am expecting the final answer to be obvious, four root five.

I hear various voices saying "four root ten," "four root five," "two root ten." So I write these numbers down and ask which is correct. There is some disagreement, but I feel that it is time to finish with notes. I tell the class that the right answer is four root five. "Okay, done," I think.

But then a hand goes up, a hand that I should be taking seriously. "Why isn't it two root ten?" I realize that if this student is unsure of this fact means I can assume that others are in the same situation. And another hand goes up. This student is asking the same thing.

I am feeling the pressure to make this answer quick, but I want to take the question seriously. And I don't know what to say. I write some numbers down and then cross them out. I don't even know what I'm saying. I try saying what I am thinking when I multiply radicals, but it doesn't seem to help.

And I see expectant faces, students who expect me to be able to explain this better. I am at a loss. I try an analogy, and show two times a monomial instead of two times the radical. Then I go back to the radical form and say it again. When I look at the class again, I see in the students' faces that my demonstration doesn't seem to be helping. I don't know what to do.

I quickly say that the answer is four root five and start telling the class what the homework is. Then I abruptly stand up and walk to the side of the room, signaling that I have no more to say. I notice the students who asked for clarification. They are still watching me. But they slowly turn their attention to their books with a look of resignation.

I feel terrible. I am an expert mathematics teacher, and I couldn't explain this little thing. I am torn between trying to explain more and resigning myself to my failure. Feeling tired, I opt for failure.

3. "Just" (week 14): I wrote this paper in response to my discussion with students about the word *just*. Though I felt like I suggested to them the possibility of being offended when a teacher used the word *just*, I was annoyed that no one defended its use. I gave copies of this paper to each student on the day following our discussion about *just*. This was in the fourteenth week.

"Just Go": Diminuendo in Mathematics Classroom Discourse

(a tentative and concise draft paper)

"And you just change it to two square root five"

The student who said this could have used the word 'simply' here instead of 'just.' Either of these words can serve as diminutives. They de-emphasize the importance or complexity of an action. When we use 'just,' we are saying, "now this is really easy." I am interested in the use of such diminutives in mathematics classrooms.

As I have observed the use of these diminutives, I first thought that they are simply insulting. When a teacher uses them, he or she is telling students that the procedures under discussion are easy, simple, routine. And I worried about the feelings of students, for whom the procedures may not be so simple. When I showed a group of students the above quotation, they came to the same conclusion. The problem is intensified because students do not generally pay attention to the language used in their classrooms. Because of their teacher's use of diminutives, they may sense their own relative inadequacy without knowing how they are getting this message.

Though it may be disturbing to notice this common way of telling students they are inadequate, grading and evaluation is a reality faced by them every day. Perhaps teachers' use of diminutives can be seen as a softer form of evaluation. We can show students where they stand relative to class expectations with these subtle messages. Instead of saying explicitly, "Solving a linear equation should be easy for you" and "Finding the relationship between the angles is probably a challenge for you right now," we might say, "Because of the properties of inscribed and central angles in circles, we know that this angle is double that one. So we can write $2(x - 15) = x + 40$, and then *just* solve that." We explain our reasoning explicitly for the challenging stuff and use diminutives to refer to the easy stuff. In this way, students can get a sense of their teacher's expectations without explicit discussion about these expectations.

But I think there are other good reasons for using diminutives. They can be a way of pointing. Consider the above example using circle geometry. If the teacher explains carefully the reasoning and procedure for solving the equation, then the students are likely to be unsure about what is important, or what is new to them. The de-emphasis of equation-solving procedures emphasizes the circle geometry reasoning. With our ways of emphasizing and de-emphasizing, we point attention to the important ideas that we are talking about.

Besides using adverbs like 'just' and 'simply' to de-emphasize, we can use verbs to do the same thing. When talking through my mathematics for others, I might just show my calculations and say, "and you go $2x - 30 = x + 40$, $x - 30 = 40$, and $x = 10$." In this case I am not saying what I am doing, I am just saying to my audience, "you to can *go* down this path, a path which should be really obvious." When I say 'just go' it is a double diminutive.

These diminutives are used very often in mathematics class. Students use them. Teachers use them. Now I have a few outstanding questions, for which I have only tentative answers:

1. Is there something special about diminutives in mathematics classes? What is it about mathematics that makes this language practice unique?
2. With issues relating to diminutives in mind, what might a mathematics teacher do differently? And, what might a student do differently?

4. Looking at the Finger (week 15): I wrote this essay to prompt students' response to the questions I raise in it. This essay also prompted Arwa and Tharshini to consider a paralinguistic aspect of communication, an aspect I was not expecting to be prompted by this essay. Our conversation about their ideas forms the basis of Chapter 6. I distributed copies of this essay to the students and Mrs. Hill at the beginning of the fifteenth week of the term. I included a copy of the cartoon referred to in the essay, but I have not been able to acquire the rights to copy it here. Here is my description of the cartoon:

> A man scolds his dog while pointing at a mess on the floor. The dog looks at the man's finger, not at the mess. The picture appears twice. The first frame is entitled "What we say to dogs." In it the man is saying, "Okay Ginger, I've had it! You stay out of the garbage! Understand, Ginger? Stay out of the garbage, or else!" In the second frame, entitled " What they hear," the man is saying, "blah blah GINGER blah blah blah blah blah blah blah blah GINGER blah blah blah blah blah." The cartoon appears in Larson (1984), which does not have page numbers.

What are we doing here?:

In this "The Far Side" cartoon, artist Gary Larson, directs our attention to peculiarities of human communication. We can only imagine what the dog is thinking. Why is she looking at the finger instead of the garbage that the man is pointing to? And how can she hope to understand when she pays attention to only one word out of the many – "Ginger"?

Oddity

Similarly, my attempts to discuss language in mathematics class have been unnatural. In some ways, I have been asking us to act like the dog. With language, we point to mathematical ideas and processes. Normally we don't think about the language, we just use it to direct each other's attention to what's really important. But when we attend to language itself, we are looking at the finger for a moment, instead of trying to see what the finger is pointing at. In this class, I have focused our attention on particular words, like "just" and "I" (not "Ginger""), as a way of drawing attention to the language (the pointing finger).

So, it should not be surprising that you, or any other students, would find it strange to talk about language in mathematics class. However, I once taught a class that regularly chatted with me about the language choices in their textbook. My students seemed to think it a natural thing to do, though I was aware that it was uncommon. So I ask myself, what is different about that previous class and this one here?

There are some obvious differences. In your class, it was clear from the beginning that we would be doing something strange. There were permission letters to be signed, there is a video recorder running, and there are two teachers, one from the university. How could you think this is normal? So every time I mention the word 'language,' we are reminded (sometimes subconsciously) that we are doing something strange, something extra.

Value

Okay. So it is strange. So what? I am interested in knowing what is actually happening when we talk about language? Of course, it is impossible to know or find the answer to this question. We cannot know how things would be different for us, if it weren't for our talks about language. And the sense of oddity I describe above makes this question all the more surreal.

When Dr. Pimm visited, I had asked him to ask you what was happening in our talks about language. I knew your answer to him would be different from your answer to me if I were to ask the same question (and I will be asking you this question below). Answering Dr. Pimm, someone said the goal is to "make it easier." Make what easier? Is it to help you do better at tests? Or is it something else? How does it change the way we see and use mathematics?

Think about mathematics as a tool for a moment. We can learn about a tool in more than one way. (Think about some tool you have learned to use recently – a blender, a drill, a gun, a computer, ...) We can learn how to use the tool – learning how it works and what we have to do to make it work. Or, we can learn how the tool is used – seeing what work people do with it. And there are others things we can learn about tools.

In every math class you've ever had, you've used language to point attention to mathematical ideas and procedures. And now, in this class, I have had you look briefly at the language – the pointing finger.

- What does this do for your understanding of mathematics?
- Does it change the way you communicate in class (your speaking, your writing, your listening)?
- Does it change the way you see your tests, or change your ability to perform well?
- Does it change the way you see mathematics relative to the rest of your subjects in school?
- Does it change the way you see the world around you?

If you have answers to these questions, I would be very interested in your responses. You can write your answers. Or, you can tell me your answers. I prefer writing, because people think about what they're saying more when they write. And if answers come to you over the holidays, please write them down and give them to me in the new year (though I'd prefer to hear your answers sooner). To clarify your descriptions, you can give an example of a particular time in which you thought about language and the way it is used.